# Revised Diagonal Method to solve assignment problem

**Anuj Kumar Katariya**[1] and **Shashi Sharma**[2]

[1,2]DAV (PG) Collège Muzaffarnagar,
C.C.S University, Meerut (U.P), India
akataria234@gmail.com

## Abstract

**We endeavor in this paper to introduce a new approach to assignment problem namely "Revised Diagonal Assignment Method (RDA)" for solving wide range of problem. We examine two numerical examples by using revised method and compute by Hungarian method. Also, we compare the optimal solutions among this revised method and the Hungarian method. The revised method is a systematic procedure, easy to apply for solving assignment problem.**

**Keywords: Assignment problem, Hungarian assignment method (HA-method), Cost minimization, linear integer programming, Revised Diagonal Assignment Method (ROA).**

## 1. Introduction

The assignment problem, one of the fundamental optimization problems, is a linear programming model which is arranged to match the resources (employee, machine etc.) with varying tasks. The key idea is to get maximum profit or sale and to get minimum cost. Since AP is a special type of transplantation problem, it needs to provide the following expressions:

- Equilibration of supply and demand (square matrix)
- Supply from every supply node is one
- Every demand node has a demand of one
- The optimal result value is an integer, such as 0 or 1

Although all the algorithms developed for transportation problem are suitable for the assignment problem, solving the assignment problem as a transportation problem is complicated and time consuming. Thus, new methods have been developed for the assignment problem

The assignment method helps in picking the "Task" and matching with the most appropriate "Agent", which can give most efficient outcome. It was first sensed by Votaw and Ordenin [3] in 1952. Application of the assignment model are numerous and widespread in our daily life. It is not just limited to assignment jobs to employees or machines; its scope is widespread. Presently, in the literature, some of the classical algorithms developed for AP solution are: Branch Boundary Algorithm, Brute Force Algorithm, and Hungarian Algorithm. There are also heuristic algorithms (genetic algorithms, parallel auction algorithm, penalty method, and greedy method) for solving this problem. Among these, the Greedy algorithm is one of the most preferred methods because it is easy to apply and requires a small number of iterations. There are more than 10 algorithms that are similar in performances (and which one is the best depends on specific situations) in these days, and studies about this topic are still on-going. Classical and heuristic algorithms for assignment problems are given in Table 1.

Table 1: Algorithms for assignment problems

| **Classical Algorithms** | **Heuristic Algorithms** |
|---|---|
| Simplex Method (LP) | OACE Algorithm |
| Transportation Method | Auction Algorithm |
| Brute Force Algorithm | Greedy Algorithm |
| The Branch Boundary Algorithm | Genetic Algorithm |
| Hungarian Algorithm | Harmony Search Algorithm |
| Lagrange Relaxation Algorithm | Penalty Method |

Among diverse approaches developed for assignment problem, Hungarian method [7], linear approaches [16], and the newer one- Auction algorithm [5] and heuristic algorithms are commonly used. The Branch Boundary Algorithm, reported by Breu and Burdet (1974), Wolsey and Nemhauser (1988), and Rijavec (1992) is the only known algorithm for solving the multidimensional assignment problem optimally [2, 14, 10]. For problems of any size, since the time required for computing the optimal solution grows exponentially with the size of the problem, this algorithm is impractical [13].

Since assignment problem in Operations Research is a main problem, many studies have been conducted about this topic and various algorithms have emerged [1]. Basirzadeh [ 6] worked and developed a Hungarian like method knows as Ones Assignment method which maximizes or minimizes the objective function. Muley and Ghadle [ 9] presented a new step in Basirzadeh's method and hence named it revised Ones assignment method. H. Gamel [11] also brought to light the drawbacks of One assignment method with some modification. Although these algorithms [12-14] are efficient in general, they possess drawbacks which can lead to poor performance. In this study, revised version of the Diagonal approach will be present to attain an exact optimal solution. Besides, the proposed algorithm will overcome the drawbacks proposed method

## 2. Mathematical formulation of an Assignment Problem:

Suppose there are n-jobs for a factory and has n-machines to process those jobs. A job i (i = 1,2, 3, ……., n) when processed by machine j ( j =1,2,3,….., n) is assumed to incur a cost $c_{ij}$ . The assignment is to be made in such a way that each job can be associated with one and only one machine. Determine an assignment of jobs to machines to minimize the overall cost.

Suppose $x_{ij}$ denote the assignment of i$^{th}$ resources to j$^{th}$ activity such that

$$x_{ij} = \begin{cases} 1, & if\ the\ i^{th}\ job\ is\ assign\ to\ the\ j^{th}\ machine \\ 0, & if\ the\ i^{th}\ job\ is\ not\ assign\ to\ j^{th}\ machine \end{cases}$$

Then the mathematical formulation of the assignment problem is,

Minimize $Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to:

$\sum_{i=1}^{n} x_{ij} = 1$ and $\sum_{j=1}^{n} x_{ij} = 1$

and $x_{ij}$ is either zero or one for all i, j = 1,2,3,…….n.

## 3. The Algorithm Is as Follows:

1). Locate the cell that have minimum cost and make assignment to the cell and delete the corresponding row and column repeat this step until all assignment are made, if the minimum element occur more than once than go to step 2 otherwise go to step 3

2). If there is a tie of minimum elements:

2.1) In any column, then Locate the two cells that have minimum cost and next to minimum cost In each row, then write their difference along the side of the table against the corresponding row and make assignment to the row which has maximum difference.

2.2) In any row then Locate the two cells that have minimum cost and next to minimum cost In each column in which the minimum elements occurs, then write their difference below the corresponding column and make assignment to the column which has maximum difference

2.3) If the difference corresponding to two or more rows/columns are equal, find the difference between first and third minimum value. Identify the maximum among them and assign the minimum cost among them.

***Remark***: The maximization problem is converted into minimization problem by subtracting from the highest sales value with all elements of the given cost matrix.

Repeat step 1 and 2 until each column have assignment, this gives the initial solution.

For finding the optimal solution, we will follow step 4 and 5

3) Write these assigned costs on the top of the column of original assignment problem. Let $a_j$ be the assigned cost for column j. Subtract $a_j$ from each entry $c_{ij}$ of the corresponding column of assignment matrix.

4) Construct a rectangle in such a way that one corner contains negative penalty and remaining two corners are allocated to the assigned cost values in corresponding row and column. Calculate the sum of extreme cells of unassigned diagonal, say $d_{ij}$ . Locate for all $d_{ij} < 0$. Identify the most negative $d_{ij}$(most negative) and exchange the assigned cell of diagonals. Continue the process until all negative (positive) penalties are resolved.

*Remarks*: If any $d_{ij}=0$, then exchange the cells of diagonals at the end.

## 4. Numerical Examples to Illustrate the Method

**Example1: Solve the following balanced assignment problem using revised diagonal Method.**

Consider the problem of assigning five jobs to five persons. The assignment costs are given below.

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
|---|---|---|---|---|---|
| ***A*** | 17 | 11 | 8 | 16 | 20 |
| ***B*** | 9 | 7 | 12 | 6 | 15 |
| ***C*** | 13 | 16 | 15 | 12 | 16 |
| ***D*** | 21 | 24 | 17 | 28 | 26 |
| ***E*** | 14 | 10 | 12 | 11 | 13 |

Determine the optimum assignment schedule and minimum assignment cost

***Solution***

***Step 1:***

Now the minimum element is 6 so we make assignment at 6 and delete the Brow and 4 column and get

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
|---|---|---|---|---|---|
| ***A*** | 17 | 11 | 8 | 16 | 20 |
| ***B*** | ~~9~~ | ~~7~~ | ~~12~~ | 6 | ~~15~~ |
| ***C*** | 13 | 16 | 15 | 12 | 16 |
| ***D*** | 21 | 24 | 17 | 28 | 26 |
| ***E*** | 14 | 10 | 12 | 11 | 13 |

Now, the minimum element is 8 so we make assignment at 8 and delete row A and column 3 and get

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
|---|---|---|---|---|---|
| ***A*** | ~~17~~ | ~~11~~ | 8 | ~~16~~ | ~~20~~ |
| ***B*** | ~~9~~ | ~~7~~ | ~~12~~ | 6 | ~~15~~ |
| ***C*** | 13 | 16 | 15 | 12 | 16 |
| ***D*** | 21 | 24 | 17 | 28 | 26 |
| ***E*** | 14 | 10 | 12 | 11 | 13 |

Now, the minimum element of the uncovered elements is 10 so we make assignment at 10 and crossed out the row E and column 2

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
|---|---|---|---|---|---|
| ***A*** | ~~17~~ | ~~11~~ | 8 | ~~16~~ | ~~20~~ |
| ***B*** | ~~9~~ | ~~7~~ | ~~12~~ | 6 | ~~15~~ |
| ***C*** | 13 | 16 | 15 | 12 | 16 |
| ***D*** | 21 | 24 | 17 | 28 | 26 |
| ***E*** | ~~14~~ | 10 | ~~12~~ | ~~11~~ | ~~13~~ |

Of the uncovered elements the minimum element is 13 so we make assignment at 13 and cover row C and column 1 by lines, and we left with only one uncover element 26 so we assign 26.

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
|---|---|---|---|---|---|
| ***A*** | ~~17~~ | ~~11~~ | 8 | ~~16~~ | ~~20~~ |
| ***B*** | ~~9~~ | ~~7~~ | ~~12~~ | 6 | ~~15~~ |
| ***C*** | 13 | ~~16~~ | ~~15~~ | ~~12~~ | ~~16~~ |
| ***D*** | 21 | 24 | 17 | 28 | 26 |
| ***E*** | ~~14~~ | 10 | ~~12~~ | ~~11~~ | ~~13~~ |

Now we have Assignment in each column. now using step 3 and 4 we will find if the solution is optimum or not.

Now write the assigned costs are on the top of the columns of original assignment matrix. Subtract assigned cost from each entry of the corresponding column of the assignment matrix. Identify all the negative penalties.

| 13 | 10 | 8 | 6 | 26 |
|---|---|---|---|---|
| 17 | 11 | 8 | 16 | 20 |
| 9 | 7 | 12 | 6 | 15 |
| 13 | 16 | 15 | 12 | 16 |
| 21 | 24 | 17 | 28 | 26 |
| 14 | 10 | 12 | 11 | 13 |

→

| | | | | |
|---|---|---|---|---|
| 4 | 1 | 0 | 10 | -6 |
| -4 | -3 | 4 | 0 | -11 |
| 0 | 6 | 7 | 6 | -10 |
| 8 | 14 | 9 | 22 | 0 |
| 1 | 0 | 4 | 5 | -13 |

Now, we check the values of dij by using step 4,

D21=2

| | |
|---|---|
| -4 | 0 |
| 0 | 6 |

d22=2

| | |
|---|---|
| -3 | 0 |
| 0 | 5 |

D15=3

| | |
|---|---|
| -6 | 0 |
| 0 | 9 |

d25=11

| | |
|---|---|
| -11 | 0 |
| 0 | 22 |

D35= -2

| | |
|---|---|
| -10 | 0 |
| 0 | 8 |

d55=1

| | |
|---|---|
| -13 | 0 |
| 0 | 14 |

Since we have d35= -2< 0 So, we make assignment to another corresponding diagonal element. Our new assigned matrix is given below again repeating step 3 we get

| 21 | 10 | 8 | 6 | 16 |
|---|---|---|---|---|
| 11 | 17 | 8 | 16 | 20 |
| 9 | 7 | 12 | 6 | 15 |
| 13 | 16 | 15 | 12 | 16 |
| 21 | 24 | 17 | 28 | 26 |
| 14 | 10 | 12 | 11 | 13 |

→

| | | | | |
|---|---|---|---|---|
| -10 | 7 | 0 | 10 | 4 |
| -12 | -3 | 4 | 0 | -1 |
| -8 | 6 | 7 | 6 | 0 |
| 0 | 14 | 9 | 22 | 10 |
| -7 | 0 | 4 | 5 | -3 |

Again, by step 4 we have

$d_{11}$= -1

| | |
|---|---|
| -10 | 0 |
| 0 | 9 |

$d_{21}$=10

| | |
|---|---|
| -12 | 0 |
| 0 | 22 |

$d_{31}$=2

| | |
|---|---|
| -8 | 0 |
| 0 | 10 |

$d_{51}$=7

| | |
|---|---|
| 0 | 14 |
| -7 | 0 |

$d_{22}$=2

| | |
|---|---|
| -3 | 0 |
| 0 | 5 |

$d_{25}$=6

| | |
|---|---|
| 0 | -1 |
| 6 | 0 |

$d_{55} = 3$

| | |
|---|---|
| 6 | 0 |
| 0 | -3 |

$d_{11}$<0, $d_{22}$>0, $d_{21}$>0, $d_{31}$>0, $d_{51}$>0, $d_{25}$>0, $d_{55}$>0

since $d_{11}$<0 so exchange the assigned cells of ***1*** and ***3***, so we get

| 11 | 10 | 17 | 6 | 16 |
|---|---|---|---|---|
| 11 | 17 | 8 | 16 | 20 |
| 9 | 7 | 12 | 6 | 15 |
| 13 | 16 | 15 | 12 | 16 |
| 21 | 24 | 17 | 28 | 26 |
| 14 | 10 | 12 | 11 | 13 |

Now again subtracting the assign entry from the corresponding column we get.

| | | | | |
|---|---|---|---|---|
| 0 | 7 | -9 | 10 | 4 |
| -2 | -3 | -5 | 0 | -1 |
| 2 | 6 | -2 | 6 | 0 |
| 10 | 14 | 0 | 22 | 10 |
| 3 | 0 | -5 | 5 | -3 |

$d_{13} = 1$; $d_{21} = 8$; $d_{22} = 2$; $d_{23} = 1$; $d_{25} = 5$; $d_{33} = 8$; $d_{53} = 9$; $d_{55} = 3$

Since, the sum of all unassigned diagonal cells is greater than zero, optimal solution has been successfully achieved. Hence, the minimum cost will be Min Cost = 11+6+16+17+10 = 60. and minimum cost using Hungarian method is 11+7+17+12+13=60

## Example 2:

Consider the problem of assigning five jobs to five persons. The assignment costs are given below.

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
|---|---|---|---|---|---|
| ***A*** | 12 | 8 | 7 | 15 | 4 |
| ***B*** | 7 | 9 | 1 | 14 | 10 |
| ***C*** | 9 | 6 | 12 | 6 | 7 |
| ***D*** | 7 | 6 | 14 | 6 | 10 |
| ***E*** | 9 | 6 | 12 | 10 | 6 |

Determine the optimum assignment schedule and minimum assignment cost.

***Step 1***:
The minimum element is 1 so we will be crossed out the row B and column 3 and get

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
|---|---|---|---|---|---|
| ***A*** | 12 | 8 | 7 | 15 | 4 |
| ***B*** | ~~7~~ | ~~9~~ | ~~1~~ | ~~14~~ | ~~10~~ |
| ***C*** | 9 | 6 | 12 | 6 | 7 |
| ***D*** | 7 | 6 | 14 | 6 | 10 |
| ***E*** | 9 | 6 | 12 | 10 | 6 |

Now, the minimum of the uncovered elements is 4 so we crossed out row ***A*** and column ***5*** and get

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
|---|---|---|---|---|---|
| ***A*** | ~~12~~ | ~~8~~ | ~~7~~ | ~~15~~ | ~~4~~ |
| ***B*** | ~~7~~ | ~~9~~ | ~~1~~ | ~~14~~ | ~~10~~ |
| ***C*** | 9 | 6 | 12 | 6 | 7 |
| ***D*** | 7 | 6 | 14 | 6 | 10 |
| ***E*** | 9 | 6 | 12 | 10 | 6 |

Now, the minimum of the elements is 6 which occurs more than once so we will go to ***step 2***

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***difference*** |
|---|---|---|---|---|---|---|
| ***A*** | ~~12~~ | ~~8~~ | ~~7~~ | ~~15~~ | ~~4~~ | |
| ***B*** | ~~7~~ | ~~9~~ | ~~1~~ | ~~14~~ | ~~10~~ | |
| ***C*** | 9 | 6 | 12 | 6 | 7 | 0 |
| ***D*** | 7 | 6 | 14 | 6 | 10 | 0 |
| ***E*** | 9 | 6 | 12 | 10 | 6 | 3 |

So maximum difference is 3 so we will assign in row ***E*** and cross the corresponding row and columns
And by using ***step 2*** we get

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***Difference*** |
|---|---|---|---|---|---|---|
| ***A*** | ~~12~~ | ~~8~~ | ~~7~~ | ~~15~~ | ~~4~~ | |
| ***B*** | ~~7~~ | ~~9~~ | ~~1~~ | ~~14~~ | ~~10~~ | |
| ***C*** | 9 | 6 | 12 | 6 | 7 | 3 |
| ***D*** | 7 | 6 | 14 | 6 | 10 | 1 |
| ***E*** | ~~9~~ | ~~6~~ | ~~12~~ | ~~10~~ | ~~6~~ | |

So, we make assignment in the row C and delete the corresponding row and column and get since only row D and column is left out without an assignment, we make assignment at the cell containing 7 and get

| | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
|---|---|---|---|---|---|
| ***A*** | ~~12~~ | ~~8~~ | ~~7~~ | ~~15~~ | ~~4~~ |
| ***B*** | ~~7~~ | ~~9~~ | ~~1~~ | ~~14~~ | ~~10~~ |
| ***C*** | ~~9~~ | ~~6~~ | ~~12~~ | ~~6~~ | ~~7~~ |
| ***D*** | 7 | 6 | 14 | 6 | 10 |
| ***E*** | ~~9~~ | ~~6~~ | ~~12~~ | ~~10~~ | ~~6~~ |

Now we have initial assignment, to check if the solution is optimum, we go to step 3 and 4 and by step3 we have

| 7 | 6 | 1 | 6 | 4 |
|---|---|---|---|---|
| 12 | 8 | 7 | 15 | 4 |
| 7 | 9 | 1 | 14 | 10 |
| 9 | 6 | 12 | 6 | 7 |
| 7 | 6 | 14 | 6 | 10 |
| 9 | 6 | 12 | 10 | 6 |

→

| | | | | |
|---|---|---|---|---|
| 5 | 2 | 6 | 9 | 0 |
| 0 | 3 | 0 | 8 | 6 |
| 2 | 0 | 11 | 0 | 3 |
| 0 | 0 | 13 | 0 | 6 |
| 2 | 0 | 11 | 4 | 2 |

Since all elements in the deviation matrix is positive, optimal solution is achieved given by 4+1+6+7+6=24

Also using Hungarian method, the solution is 4+1+6+7+6=24.

## 5. Analysis and Discussion:

In any minimization type Assignment problem, we must choose the combination of least elements from each row and columns. Now we can go to the initial solution by any possible method but the main problem occurs is the minimum cost, so if we subtract the initial solution elements from the corresponding columns we will get the deviation of the entries of the cost matrix. Now if all the elements in the deviation matrix is positive this means we have selected the minimum element of the columns, and the optimum solution is achieved. If we have negative entries in a column of the deviation matrix than the corresponding columns do not have the minimum assignment and optimum solution may not be achieved. So, each negative entries of the deviation matrix are to be discussed.

Now if we have a negative entry at any cell so we have to check if it is the best assignment suited for the problem or we have to change the assignment cell, as in an n*n cost matrix we can use only n entries which are already chosen. So, we have to shuffle our initial assigned cells, now if we have a negative entry in the deviation matrix than we find the assigned cells in the corresponding row and column of the negative entry. And this forms a right triangle if we assume the cells as vertex, now we complete the rectangle because we have to discuss the non-zero elements which lies on the diagonal of the rectangle with two adjacent vertices as the assigned cells

Assigned matrix

| | | | | |
|---|---|---|---|---|
| A11 | A12 | A13 | A14 | A15 |
| A21 | A22 | A23 | A24 | A25 |
| A31 | A32 | A33 | A34 | A35 |
| A41 | A42 | A43 | A44 | A45 |
| A51 | A52 | A53 | A54 | A55 |

→ Deviation matrix

| | | | | |
|---|---|---|---|---|
| (A11-A41) | + | 0 | + | + |
| - | - | + | 0 | - |
| - | + | + | + | 0 |
| 0 | + | A43-A13 | + | + |
| - | 0 | + | + | + |

let us suppose we have a 5*5 initial cost matrix and we have a negative entry at the cell A11 in the deviation matrix and the assigned cell in the 1st row and 1st column are at A13 and A41 respectively, so in the first column we have assignment in the 4th row and in 3rd column we have assignment in the 1st row so we will check if it is the optimum assignment the and if it is not then, we have to exchange these assigned cells to minimize cost. And the best way to do this is to find the sum of the deviation of the initial assignment. If the sum of the deviation is more than 0 then the total cost will be more than

the initial cost, so we will check where we have sum of deviations less than 0. As here we have sum of deviation is (A11-A41) +(A43-A13) = A11+A43 – (A41+A13) = x(say)

if x< 0 →A11+A43< A41+A13 so, our initial solution is not optimal, and we will choose A11 and A43 which are the elements on the other diagonal and the assignment cost is reduced.

and if x>0→A11+A43> A41+A13 so we have the best choice to the minimum cost, and we should not exchange the assigned cells here.

## 6. Conclusion:

In this paper, Revised Diagonal method was introduced to attain an exact optimal solution of assignment problem. The revised method is different from previously reported method in several ways. This method can be used for maximized as well as minimized or balanced and unbalanced objective functions of an assignment problem. As this method consumes less time and is very easy to understand it will be helpful for decision makers. The proposed revised Diagonal Assignment method overcomes the drawbacks of old method and outperforms them by a considerable margin. Despite having a few new features, the new revised diagonal method is fast, efficient and is equally suitable for different types of assignment problem.

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