# CONTRA REGULAR GENERALIZED AND ALMOST REGULAR GENERALIZED CONTINUOUS MAPPINGS IN BIPOLAR PYTHAGOREAN FUZZY TOPOLOGICAL SPACES

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***Abstract:*** *In this paper, we introduce Bipolar Pythagorean Fuzzy Contra Regular Generalized Continuous Mapping and Bipolar Pythagorean Fuzzy Almost Regular Generalized Continuous Mapping and study some of their basic properties.*

*Keywords:* Bipolar Pythagorean Fuzzy sets, Bipolar Pythagorean Fuzzy Topology, Bipolar Pythagorean Fuzzy Regular Generalized Closed sets, Bipolar Pythagorean Fuzzy Contra Regular Generalized Continuous Mapping and Bipolar Pythagorean Fuzzy Almost Regular Generalized Continuous Mapping.

## 1. INTRODUCTION

Fuzzy set is introduced by Zadeh [1]. After that, Atanassov [2] introduced the notion of intuitionistic fuzzy sets and Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. After that, Yager [4] proposed another class of nonstandard fuzzy sets, called Pythagorean fuzzy sets and Murat Olgun, Mehmet Ünver, Seyhmus Yardimci [5] introduced the notion of Pythagorean fuzzy topological spaces and Zhang [6] introduced the extension of fuzzy set with bipolarity, called Bipolar value fuzzy sets. Bosc and Pivert [10] said that "Bipolarity" refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative effects. Positive information states what is possible, satisfactory, permitted, desired or considered as acceptable. Negative statement corresponds to what is impossible, rejected or forbidden. Negative preferences to constraints, since they specify which values or objects have to be rejected, while positive preferences correspond to wishes, as they specify which objects are more desirable than others, without rejecting those that do not meet the wishes. In bipolar valued fuzzy set interval of membership value is [-1,1].

In this paper, we introduce, Bipolar Pythagorean Fuzzy Contra Regular Generalized Continuous Mapping and Bipolar Pythagorean Fuzzy Almost Regular Generalized Continuous Mapping and study some of their basic properties.

## 2. PRELIMINARIES

**Definition 2.1:** Let X be the non empty universe of discourse. A fuzzy set A in X, $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set A; $\mu_A(x) \in [0,1]$ is the membership of $x \in X$.

**Definition 2.2:** Let X be the non empty universe of discourse. An Intuitionistic fuzzy set(IFS) A in X is given by A=$\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A(x) \in [0,1]$

and $\nu_A(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$to the set A, respectively, and $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$ for each $x \in X$.

The degree of indeterminacy $I_A = 1 - (\mu_A(x) - \nu_A(x))$ for each $x \in X$.

**Definition 2.3:** Let X be the non empty universe of discourse. A Pythagorean fuzzy set(PFS) P in X is given by P=$\{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$ where the functions $\mu_P(x) \in [0,1]$ and $\nu_P(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set P, respectively, and $0 \leqslant \mu_P^2(x) + \nu_P^2(x) \leqslant 1$ for each $x \in X$.

The degree of indeterminacy $I_P = \sqrt{1 - \mu_P^2(x) - \nu_P^2(x)}$ for each $x \in X$.

**Definition 2.4:** Let X be a non empty set. A Bipolar Pythagorean Fuzzy Set $A = \{(x, \mu_A^+, \mu_A^-, \nu_A^+, \nu_A^-) : x \in X\}$ where $\mu_A^+ : X \to [0,1], \nu_A^+ : X \to [0,1], \mu_A^- : X \to [-1,0], \nu_A^- : X \to [-1,0]$ are the mappings such that $0 \leqslant (\mu_A^+(x))^2 + (\nu_A^+(x))^2 \leqslant 1$ and $-1 \leqslant (\mu_A^-(x))^2 + (\nu_A^-(x))^2 \leqslant 0$ where

$\mu_A^+(x)$ denote the positive membership degree.

$\nu_A^+(x)$ denote the positive non membership degree.

$\mu_A^-(x)$ denote the negative membership degree.

$\nu_A^-(x)$ denote the negative non membership degree.

**Definition2.5:** Let $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B^+(x), \nu_B^+(x), \mu_B^-(x), \nu_B^-(x)\rangle : x \in X\}$ be two Bipolar Pythagorean Fuzzy sets over X. Then

i. The Bipolar Pythagorean fuzzy Complement of $A$ is defined by

$A^c = \{\langle x, \nu_A^+(x), \mu_A^+(x), \nu_A^-(x), \mu_A^-(x)\rangle : x \in X\}$,

ii. The Bipolar Pythagorean fuzzy intersection of $A$ and $B$ is defined by

$A \cap B =$

$\{\langle x, min\{\mu_A^+(x), \mu_B^+(x)\}, max\{\nu_A^+(x), \nu_B^+(x)\}, max\{\mu_A^-(x), \mu_B^-(x)\}, min\{\nu_A^-(x), \nu_B^-(x)\}\rangle : x \in X\}$

iii. The Bipolar Pythagorean fuzzy union of $A$ and $B$ is defined by

$A \cup B =$

$\{\langle x, max\{\mu_A^+(x), \mu_B^+(x)\}, min\{\nu_A^+(x), \nu_B^+(x)\}, min\{\mu_A^-(x), \mu_B^-(x)\}, max\{\nu_A^-(x), \nu_B^-(x)\}\rangle : x \in X\}$

iv. $A$ is a Bipolar Pythagorean subset of $B$ and write $A \subseteq B$ if $\mu_A^+(x) \leqslant \mu_B^+(x), \nu_A^+(x) \geqslant \nu_B^+(x), \mu_A^-(x) \geqslant \mu_B^-(x), \nu_A^-(x) \leqslant \nu_B^-(x)$ $foreach$ $x \in X$.

v. $0_X = \{\langle x, 0,1,0, -1\rangle : x \in X\}$ and $1_X = \{\langle x, 1,0, -1,0\rangle : x \in X\}$.

**Definition 2.6:** Bipolar Pythagorean Fuzzy Topological Spaces: Let $X \neq \emptyset$ be a set and $\tau_p$ be a family of Bipolar Pythagorean fuzzy subsets of X. If

$T_1$ $0_X, 1_X \in \tau_p$.

$T_2$ For any $P_1, P_2 \in \tau_p$, we have $P_1 \cap P_2 \in \tau_p$.

$T_3$ $\cup P_i \in \tau_p$ for arbitrary family$\{P_i \ni i \in J\} \subseteq \tau_p$.

Then $\tau_p$ is called Bipolar Pythagorean Fuzzy Topology on X and the pair $(X, \tau_p)$ is said to be Bipolar Pythagorean Fuzzy Topological space. Each member of $\tau_p$ is called Bipolar Pythagorean fuzzy open set(BPFOS). The complement of a Bipolar Pythagorean Fuzzy open set is called a Bipolar Pythagorean fuzzy Closed set(BPFCS).

**Definition 2.7:** Let $(X, \tau_p)$ be a BPFTS and $P = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ be a BPFS over X. Then the Bipolar Pythagorean Fuzzy Interior, Bipolar Pythagorean Fuzzy Closure of P are defined by:

a) BPF$int(P)$ =$\cup${G / G is a BPFOS in $(X, \tau_p)$ and G $\subseteq$ P}.

b) BPF$cl(P)$ =$\cap${K / K is a BPFCS in $(X, \tau_p)$ and P$\subseteq$ K}.

It is clear that

a) BPF$int(P)$ is the biggest Bipolar Pythagorean Fuzzy Open set contained in P.

b) BPF$cl(P)$ is the smallest Bipolar Pythagorean Fuzzy Closed set containing P.

**Definition 2.8.** If BPFS $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ in a BPTS $(X, \tau_p)$ is said to be
(a) Bipolar Pythagorean Fuzzy Semi closed set (BPFSCS) if $int(cl(A)) \subseteq A$.
(b) Bipolar Pythagorean Fuzzy Semi open set (BPFSOS) if $A \subseteq cl(int(A))$.
(c) Bipolar Pythagorean Fuzzy Preclosed set (BPFPCS) if $cl(int(A)) \subseteq A$.
(d) Bipolar Pythagorean Fuzzy Preopen set (BPFPOS) if $A \subseteq int(cl(A))$.
(e) Bipolar Pythagorean Fuzzy $\alpha$ closed set (BPF$\alpha$CS) if $cl(int(cl(A)) \subseteq A$.
(f) Bipolar Pythagorean Fuzzy $\alpha$ open set (BPF$\alpha$OS) if $A \subseteq int(cl(int(A))$.
(g)Bipolar Pythagorean Fuzzy $\gamma$ closed set (BPF$\gamma$CS) if $A \subseteq int(cl(A) \cup cl(int(A))$.
(h)Bipolar Pythagorean Fuzzy $\gamma$ open set (BPF$\gamma$OS) if $cl(int(A) \cup int(cl(A)) \subseteq A$.
(i) Bipolar Pythagorean Fuzzy regular closed set (BPFRCS) if $A = cl(int(A))$.
(j) Bipolar Pythagorean Fuzzy regular open set (BPFROS) if $A = int(cl(A))$.
(k) If BPF set $A$ of a BPFTS $(X, \tau_p)$ is a Bipolar Pythagorean Fuzzy Generalized closed set (BPFGCS), if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is BPFOS in $(X, \tau_p)$.
(l) If BPF set $A$ of a BPFTS $(X, \tau_p)$ is a Bipolar Pythagorean Fuzzy Generalized open set (BPFGCS), if $A^c$ is a BPFGCS in $(X, \tau_p)$.

**Definition 2.9:** A Bipolar Pythagorean Fuzzy Set A of a Bipolar Pythagorean Fuzzy Topological Space $(X, \tau_p)$ is called Bipolar Pythagorean Regular Generalized closed (BPFRGCS in short), if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is BPF regular Open in $(X, \tau_p)$.

**Definition 2.10:** A Bipolar Pythagorean Fuzzy Set $A$ of a Bipolar Pythagorean Fuzzy Topological Space $(X, \tau_p)$ is called Bipolar Pythagorean Regular Generalized Open (BPFRGOS in short), if $int(A) \supseteq U$ whenever $A \supseteq U$ and $U$ is BPFR-closed in $(X, \tau_p)$.

**Definition 2.11:** Let $f$ be a mapping from an BPFTS in $(X, \tau_p)$ into a BPFTS $(Y, \sigma_p)$. Then $f$ is said to be Bipolar Pythagorean Fuzzy Continuous mapping if $f^{-1}(A) \in BPFO(X)$ for every $A \in (Y, \sigma_p)$.

**Definition 2.12:** A mapping $f : (X, \tau_p) \to (Y, \sigma_p)$ is said to be
(i) BPF semi continuous mapping if $f^{-1}(A) \in$ BPFSO(X) for every $A \in (Y, \sigma_p)$.
(ii) BPF$\alpha$ continuous mapping if $f^{-1}(A) \in$ BPF$\alpha$O(X) for every $A \in (Y, \sigma_p)$.
(ii) BPF Pre continuous mapping if $f^{-1}(A) \in$ BPFPO(X) for every $A \in (Y, \sigma_p)$.
(ii) BPF$\gamma$ continuous mapping if $f^{-1}(A) \in$ BPF$\gamma$O(X) for every $A \in (Y, \sigma_p)$.

**Definition 2.13:** A mapping $f : (X, \tau_p) \to (Y, \sigma_p)$ is said to be BPF Generalized Continuous mapping (BPFG continuous mapping) if $f^{-1}(A) \in$ BPFGC(X) for every BPFCS $A$ in $(Y, \sigma_p)$.

**Definition 2.14:** A mapping $f : (X, \tau_p) \to (Y, \sigma_p)$ is said to be BPF $\alpha$ Generalized Continuous mapping (BPF$\alpha$G continuous mapping) if $f^{-1}(A) \in$ BPF$\alpha$C(X) for every BPFCS $A$ in $(Y, \sigma_p)$.

**Definition 2.15:** A BPFTS $(X, \tau_p)$ is said to be a $BPFRG_cT_{1/2}$ space (Bipolar Pythagorean Fuzzy Regular Generalized ${}_cT_{1/2}$ space) if every BPFRGCS in $(X, \tau_p)$ is a BPFCS in $(X, \tau_p)$.

**Definition 2.16:** A BPFTS $(X, \tau_p)$ is said to be a $BPFRG_gT_{1/2}$ space (Bipolar Pythagorean Fuzzy Regular Generalized ${}_gT_{1/2}$ space) if every BPFRGCS in $(X, \tau_p)$ is a BPFGCS in $(X, \tau_p)$.

**Definition 2.17:** A BPFTS $(X, \tau_p)$ is said to be a $BPFRG_\alpha T_{1/2}$ space (Bipolar Pythagorean Fuzzy Regular Generalized ${}_\alpha T_{1/2}$ space) if every BPFRGCS in $(X, \tau_p)$ is a BPF$\alpha$CS in $(X, \tau_p)$.

**Definition 2.18:** A mapping $f: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra continuous mapping (BPF contra continuous mapping) if $f^{-1}(A) \in$ BPFOS(X) for every BPFCS $A$ in $(Y, \sigma_p)$.

**Definition 2.19:** A mapping $f: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra $\alpha$ continuous mapping (BPF contra $\alpha$ continuous mapping) if $f^{-1}(A) \in$ BPF$\alpha$OS(X) for every BPFCS $A$ in $(Y, \sigma_p)$.

**Definition 2.20:** A mapping $f: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra Pre continuous mapping (BPF contra Pre continuous mapping) if $f^{-1}(A) \in$ BPFPOS(X) for every BPFCS $A$ in $(Y, \sigma_p)$.

**Definition 2.21** A mapping $f: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra semi continuous mapping (BPF contra semi continuous mapping) if $f^{-1}(A) \in$ BPFSOS(X) for every BPFCS $A$ in $(Y, \sigma_p)$.

**Definition 2.22:** A mapping $f: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra Generalized continuous mapping (BPF contra G continuous mapping) if $f^{-1}(A) \in$ BPFGOS(X) for every BPFCS $A$ in $(Y, \sigma_p)$.

**Definition 2.23:** A mapping $f: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra $\alpha$ G continuous mapping (BPF contra $\alpha$ G continuous mapping) if $f^{-1}(A) \in$ BPFP$\alpha$GOS(X) for every BPFCS $A$ in $(Y, \sigma_p)$.

# 3 BIPOLAR PYTHAGOREAN FUZZY CONTRA REGULAR GENERALIZED CONTINUOUS MAPPING (BPF$C$RG CONTINUOUS MAPPING)

In this section we are going to study the new concept of contra regular generalized continuous mapping in Bipolar Pythagorean Fuzzy Topological space.

**Definition 3.1:** A mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Contra Regular Generalized Continuous mapping (BPF$C$RG Continuous mapping) if $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$ for every BPFOS $D$ in $(Y, \sigma_p)$.

**Example 3.2:** Let X={a,b}, Y={u,v}, $T_1$=(x, (0.8, 0.6), (0.5, 0.4), (-0.7, -0.6), (-0.3, -0.2)), $T_2$=(x, (0.5, 0.4), (0.8, 0.6), (-0.3, -0.2), (-0.7, -0.7)) and $T_3$=(y, (0.4, 0.5), (0.7, 0.6), (-0.4, -0.6), (-0.4, -0.2)). Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, 1_p\}$ are BPFTs on $X$ and $Y$ respectively. Define a mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ by $\kappa(a) = u$ and $\kappa(b) = v$. Here $T_3$ = (y, (0.4, 0.5), (0.7, 0.6), (-0.4, -0.6), (-0.4, -0.2)) is BPFOS in $(Y, \sigma_p)$ and $T_1$, $T_2$ are BPFROS in $(X, \tau_p)$. Now $\kappa^{-1}(T_3)$ = (y, (0.4, 0.5), (0.7, 0.6), (-0.4, -0.6), (-0.4, -0.2)) is a BPFRGCS in $(X, \tau_p)$, as $cl(\kappa^{-1}(T_3)) = T_2^c \subseteq T_1$ whenever, $\kappa^{-1}(T_3) \subseteq T_1$. Therefore $\kappa$ is said to be BPF$C$RG continuous in $(X, \tau_p)$.

**Theorem 3.3:** Every BPF$C$ continuous mapping is a BPF$C$RG continuous mapping but not conversely.

**Proof :** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF$C$ continuous mapping. Let $D$ be a BPFOS in $(Y, \sigma_p)$. By hypothesis, $\kappa^{-1}(D)$ is a BPFCS in $(X, \tau_p)$. Since every BPFCS is a BPFRGCS, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$C$RG continuous mapping.

**Example 3.4:** In Example 3.2, $\kappa^{-1}(T_3)$ = (y, (0.4, 0.5), (0.7, 0.6), (-0.4, -0.6), (-0.4, -0.2)) is BPFRGCS in $(X, \tau_p)$ but not BPFCS in $(X, \tau_p)$. Therefore, $\kappa$ is a BPF$C$RG continuous mapping in $(X, \tau_p)$ but not BPF $C$ continuous mapping in $(X, \tau_p)$, since $cl(\kappa^{-1}(T_3)) = T_2^c \neq \kappa^{-1}(T_3)$.

**Theorem 3.5:** Every BPF$C$R continuous mapping is a BPF$C$RG continuous mapping but not conversely.

**Proof :** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF$C$ continuous mapping. Let $D$ be a BPFOS in $(Y, \sigma_p)$. By hypothesis, $\kappa^{-1}(D)$ is a BPFCS in $(X, \tau_p)$. Since every BPFCS is a BPFRGCS, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$C$RG continuous mapping.

**Example 3.6:** In Example 3.2, $\kappa^{-1}(T_3)$ = (y, (0.4, 0.5), (0.7, 0.6), (-0.4, -0.6), (-0.4, -0.2)) is BPFRGCS in $(X, \tau_p)$ but not BPFRCS in $(X, \tau_p)$. Therefore, $\kappa$ is a BPF$C$RG continuous mapping in $(X, \tau_p)$ but not BPF $C$ R continuous mapping in $(X, \tau_p)$, Since $cl(int(\kappa^{-1}(T_3))) = T_2^c \neq \kappa^{-1}(T_3)$.

**Theorem 3.7:** Every BPF$C$G continuous mapping is a BPF$C$RG continuous mapping but not conversely.

**Proof:** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF$C$G continuous mapping. Let $D$ be a BPFOS in $(Y, \sigma_p)$. By hypothesis, $\kappa^{-1}(D)$ is a BPFGCS in $(X, \tau_p)$. Since every BPFGCS is a BPFRGCS, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$C$RG continuous mapping.

**Example 3.8:** In Example 3.2, $\kappa^{-1}(T_3)$ = (y, (0.4, 0.5), (0.7, 0.6), (-0.4, -0.6), (-0.4, -0.2)) is BPFRGCS in $(X, \tau_p)$ but not BPFGCS in $(X, \tau_p)$. Therefore, $\kappa$ is a BPF$C$RG continuous mapping in $(X, \tau_p)$ but not BPF$C$G continuous mapping in $(X, \tau_p)$, since $cl(\kappa^{-1}(T_3)) = T_2^c$ whenever $\kappa^{-1}(T_3) \subseteq U$, $U$ is BPF open in $(X, \tau_p)$.

**Theorem 3.9:** Every BPF$C\alpha$ continuous mapping is a BPF$C$RG continuous mapping but not conversely.

**Proof:** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF$C$ continuous mapping. Let $D$ be a BPFOS in $(Y, \sigma_p)$. By hypothesis, $\kappa^{-1}(D)$ is a BPF$\alpha$CS in $(X, \tau_p)$. Since every BPF$\alpha$CS is a BPFRGCS, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$C\alpha$RG continuous mapping.

**Example 3.10:** In Example 3.2, $\kappa^{-1}(T_3)$ = (y, (0.4, 0.5), (0.7, 0.6), (-0.4, -0.6), (-0.4, -0.2)) is BPFRGCS in $(X, \tau_p)$ but not BPF$\alpha$CS in $(X, \tau_p)$. Therefore, $\kappa$ is a BPF$C$RG continuous mapping in $(X, \tau_p)$ but not BPF$C\alpha$ continuous mapping in $(X, \tau_p)$, since $cl(int(cl(\kappa^{-1}(T_3)))) = T_2^c \neq \kappa^{-1}(T_3)$.

**Theorem 3.11:** Every BPF$C\alpha$G continuous mapping is a BPF$C$RG continuous mapping but not conversely.

**Proof:** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF$C\alpha$G continuous mapping. Let $D$ be a BPFOS in $(Y, \sigma_p)$. By hypothesis, $\kappa^{-1}(D)$ is a BPF$\alpha$GCS in $(X, \tau_p)$. Since every BPF$\alpha$GCS is a BPFRGCS, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$C$RG continuous mapping.

**Example 3.12:** In Example 3.2, $\kappa^{-1}(T_3)$ = (y, (0.4, 0.5), (0.7, 0.6), (-0.4, -0.6), (-0.4, -0.2)) is BPFRGCS in $(X, \tau_p)$ but not BPF$\alpha$CS in $(X, \tau_p)$. Therefore, $\kappa$ is a BPF$C$RG continuous mapping in $(X, \tau_p)$ but not BPF$C\alpha$G continuous mapping in $(X, \tau_p)$, since $cl(int(cl(\kappa^{-1}(T_3)))) = T_2^c \not\subseteq U$, whenever $\kappa^{-1}(T_3) \subseteq U$ and $U$ is BPF open in $(X, \tau_p)$.

**Remark 3.13:** BPF$C$P continuous mapping and BPF$C$RG continuous mapping are independent of eachother.

**Example 3.14:** Let X={a,b}, Y={u,v}, $T_1$=(x, (0.7, 0.5), (0.4, 0.3), (-0.6, -0.5), (-0.2, -0.1)), $T_2$=(x, (0.4, 0.3), (0.7, 0.5), (-0.2, -0.1), (-0.6 -0.6)) and $T_3$=(y, (0.3, 0.4), (0.6, 0.5), (-0.3, -0.5), (-0.3, -0.1)). Then $\tau_p$=$\{0_p, T_1, 1_p\}$ and $\sigma_p$=$\{0_p, T_2, 1_p\}$ are BPFTs on $X$ and $Y$ respectively. Define a mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ by $\kappa(a) = u$ and $\kappa(b) = v$. Let us consider the BPFOS $T_3$=(y, (0.3, 0.4), (0.6, 0.5), (-0.3, -0.5), (-0.3, -0.1)) in $(Y, \sigma_p)$. Here $T_1$, $T_2$ are BPFROS in $(X, \tau_p)$ and $\kappa^{-1}(T_3)$ = (x, (0.3, 0.4), (0.6, 0.5), (-0.3, -0.5), (-0.3, -0.1)) is a BPFRGCS in $(X, \tau_p)$ but $\kappa^{-1}(T_3)$ is not a BPFPCS in $(X, \tau_p)$, since $cl(int(\kappa^{-1}(T_3))) = T_2^c \not\subseteq \kappa^{-1}(T_3)$. Therefore $\kappa$ is a BPF$\mathcal{C}$RG continuous mapping but not a BPF$\mathcal{C}$P continuous mapping.

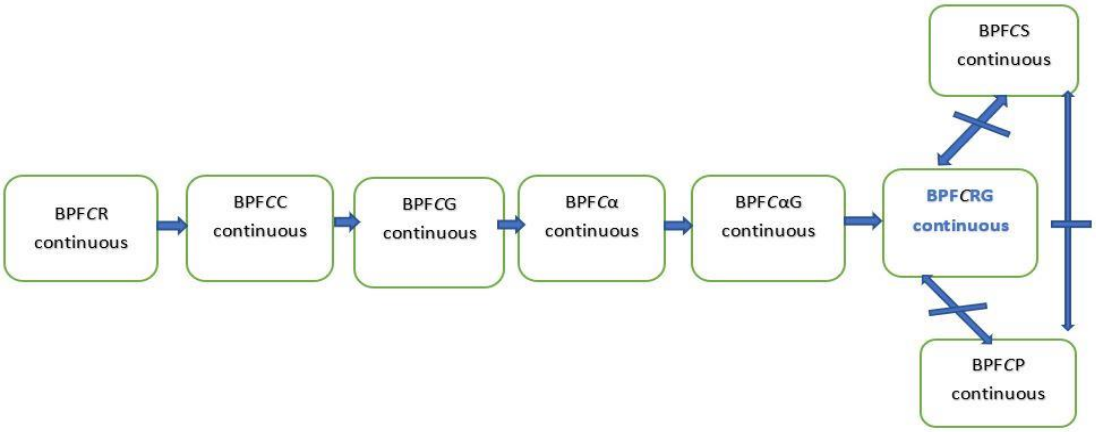
**Example 3.15:** Let X={a,b}, Y={u,v}, $T_1$=(x, (0.5, 0.3), (0.3, 0.3), (-0.5, -0.4), (-0.3, -0.3)), $T_2$=(x, (0.2, 0.3), (0.5, 0.3), (-0.2, -0.2), (-0.5, -0.8)) and $T_3$=(y, (0.2, 0.3), (0.8, 0.7), (-0.2, -0.2), (-0.8, -0.6)). Then $\tau_p$=$\{0_p, T_1, T_2, 1_p\}$ and $\sigma_p$=$\{0_p, T_3, 1_p\}$ be a BPFTs on $X$ and $Y$ respectively. Define a mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ by $\kappa(a) = u$ and $\kappa(b) = v$. Let us consider the BPFOS $T_3$=(Y, (0.2, 0.3), (0.8, 0.7), (-0.2, -0.2), (-0.8, -0.6)) in Y. Here $T_1$, $T_2$ are BPFROS in $(X, \tau_p)$ and $\kappa^{-1}(T_3)$ = (x, (0.2, 0.3), (0.8, 0.7), (-0.2, -0.2), (-0.8, -0.6)) is a BPFPCS in $(X, \tau_p)$, since $cl(int(\kappa^{-1}(T_3))) = 0_p \subseteq \kappa^{-1}(T_3)$ but $\kappa^{-1}(T_3)$ is not a BPFRGCS in $(X, \tau_p)$, Therefore $\kappa$ is not a BPF$\mathcal{C}$RG continuous mapping but BPF$\mathcal{C}$P continuous mapping.

**Remark 3.16:** BPF$\mathcal{C}$S continuous mapping and BPF$\mathcal{C}$RG continuous mapping are independent of each other.

**Example 3.17:** Let X={a,b}, Y={u,v}, $T_1$=(x, (0.7, 0.5), (0.4, 0.3), (-0.6, -0.5), (-0.2, -0.1)), $T_2$=(x, (0.4, 0.3), (0.7, 0.5), (-0.2, -0.1), (-0.6 -0.6)) and $T_3$=(y, (0.3, 0.4), (0.6, 0.5), (-0.3, -0.5), (-0.3, -0.1)). Then $\tau_p$=$\{0_p, T_1, 1_p\}$ and $\sigma_p$=$\{0_p, T_2, 1_p\}$ are BPFTs on $X$ and $Y$ respectively. Define a mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ by $\kappa(a) = u$ and $\kappa(b) = v$. Let us consider the BPFOS $T_3$=(y, (0.3, 0.4), (0.6, 0.5), (-0.3, -0.5), (-0.3, -0.1)) in $(Y, \sigma_p)$. Here $T_1$, $T_2$ are BPFROS in $(X, \tau_p)$ and $\kappa^{-1}(T_3)$ = (x, (0.3, 0.4), (0.6, 0.5), (-0.3, -0.5), (-0.3, -0.1)) is a BPFRGCS in $(X, \tau_p)$, but $\kappa^{-1}(T_3)$ is not a BPFSCS in $(X, \tau_p)$, since $int(cl(\kappa^{-1}(T_3))) = T_2^c \not\subseteq \kappa^{-1}(T_3)$. Therefore $\kappa$ is a BPF$\mathcal{C}$RG continuous mapping but not a BPF$\mathcal{C}$S continuous mapping.

**Example 3.18:** Let X={a,b}, Y={u,v}, $T_1$=(x, (0.7, 0.3), (0.7, 0.7), (-0.5, -0.3), (-0.5, -0.7)), $T_2$=(x, (0.3, 0.2), (0.7, 0.8), (-0.2, -0.2), (-0.8 -0.8)) and $T_3$=(y, (0.7, 0.3), (0.7, 0.7), (-0.5, -0.3), (-0.5, -0.7)). Then $\tau_p$=$\{0_p, T_1, 1_p\}$ and $\sigma_p$=$\{0_p, T_2, 1_p\}$ are BPFTs on $X$ and $Y$ respectively. Define a mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ by $\kappa(a) = u$ and $\kappa(b) = v$. Let us consider the BPFOS $T_3$=(y, (0.7, 0.3), (0.7, 0.7), (-0.5, -0.3), (-0.5, -0.7)) in $(Y, \sigma_p)$. Here $T_1$, $T_2$ are BPFROS in $(X, \tau_p)$ and $\kappa^{-1}(T_3)$ = (x, (0.7, 0.3), (0.7, 0.7), (-0.5, -0.3), (-0.5, -0.7)) is a BPFSCS in $(X, \tau_p)$, since $int(cl(\kappa^{-1}(T_3))) = T_1 \subseteq \kappa^{-1}(T_3)$ but not a BPFRGCS in $(X, \tau_p)$. Therefore $\kappa$ is not a BPF$\mathcal{C}$RG continuous mapping but a BPF$\mathcal{C}$S continuous mapping.

Figure 1: Relation between BPF$C$RG continuous and other existed BPF continuities




**Theorem 3.19:** A mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ is a BPFRG continuous mapping if and only if the inverse image of each BPFCS in $(Y, \sigma_p)$ is a BPFRGOS in $(X, \tau_p)$.
**Proof: Necessity:** Let $D$ be a BPFCS in $(Y, \sigma_p)$. This implies $D^c$ is a BPFOS in $(Y, \sigma_p)$. Since $\kappa$ is a BPFRG continuous mapping, $\kappa^{-1}(D^c)$ is a BPFRGCS in $(X, \tau_p)$. Since $\kappa^{-1}(D^c) = (\kappa^{-1}(D))^c$, $\kappa^{-1}(D)$ is an BPFRGOS in $(X, \tau_p)$.
**Sufficiency:** Let $D$ be a BPFOS in $(Y, \sigma_p)$. This implies $D^c$ is a BPFCS in $(Y, \sigma_p)$. By hypothesis, $\kappa^{-1}(D^c)$ is a BPFRGOS in $(X, \tau_p)$. Since $\kappa^{-1}(D^c) = (\kappa^{-1}(D))^c$, where $(\kappa^{-1}(D))^c$ is a BPFRGOS in $(X, \tau_p)$, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPFRG continuous mapping.

**Theorem 3.20:** If $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping and let $\kappa^{-1}(D)$ be a BPFROS in $(X, \tau_p)$ for every BPFCS $D$ in $(Y, \sigma_p)$. Then $\kappa$ is a BPF$C$RG continuous mapping.
**Proof:** Let $D$ be a BPFCS in $(Y, \sigma_p)$. Then $\kappa^{-1}(D)$ is a BPFROS in $(X, \tau_p)$, by hypothesis. Since every BPFROS is a BPFRGOS, $\kappa^{-1}(D)$ is a BPFRGOS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$C$RG continuous mapping,by the above theorem.

**Theorem 3.21:** If $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ is a BPF $C$ RG continuous mapping and $\chi: (Y, \sigma_p) \to (Z, \zeta_p)$ is a BPF continuous mapping, then $\chi o \kappa: (X, \tau_p) \to (Z, \zeta_p)$ is a BPF$C$RG continuous mapping.
**Proof:** Let $D$ be a BPFOS in $(Z, \zeta_p)$. Then $\chi^{-1}(D)$ is a BPFOS in $(Y, \sigma_p)$, by hypothesis. Since $\kappa$ is a BPF$C$RG continuous mapping, $\kappa^{-1}(\chi^{-1}(D))$ is a BPFRGCS in $(X, \tau_p)$. Hence $\chi o \kappa$ is a BPF$C$RG continuous mapping.

**Theorem 3.22:** If $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ is a BPF $C$ RG continuous mapping and $\chi: (Y, \sigma_p) \to (Z, \zeta_p)$ is a BPFG continuous mapping and $(Y, \sigma_p)$ is a $BPFT_{1/2}$ space, then $\chi o \kappa: (X, \tau_p) \to (Z, \zeta_p)$ is a BPF$C$RG continuous mapping.
**Proof:** Let $D$ be a BPFOS in $(Z, \zeta_p)$. Then $\chi^{-1}(D)$ is a BPFOS in $(Y, \sigma_p)$, by hypothesis. Since $(Y, \sigma_p)$ is a $BPFT_{1/2}$ space, $\chi^{-1}(D)$ is a BPFOS in $(X, \tau_p)$. Therefore $\kappa^{-1}(\chi^{-1}(D))$ is a BPFRGCS in $(X, \tau_p)$, by hypothesis. Hence $\chi o \kappa$ is a BPF$C$RG continuous mapping.

**Theorem 3.23:** If $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ is a mapping that satisfies any one of the following properties,

(i) $\kappa(cl(D)) \subseteq int(\kappa(D))$ for each BPFS $D$ in $(X, \tau_p)$
(ii) $cl(\kappa^{-1}(E)) \subseteq \kappa^{-1}(int(E))$ for each BPFS $E$ in $(Y, \sigma_p)$
(iii) $\kappa^{-1}(cl(E)) \subseteq int(\kappa^{-1}(E))$ for each BPFS $E$ in $(Y, \sigma_p)$
then $\kappa$ is a BPF$C$RG continuous mapping.

**Proof:** (i) implies (ii) Let $E$ be a BPFS in $(Y, \sigma_p)$. Then $\kappa^{-1}(E)$ is a BPFS in $(X, \tau_p)$. By hypothesis, $\kappa(cl(\kappa^{-1}(E))) \subseteq int(\kappa(\kappa^{-1}(E))) \subseteq int(E)$.
Now $cl(\kappa^{-1}(E)) \subseteq \kappa^{-1}(\kappa(cl(\kappa^{-1}(E)))) \subseteq \kappa^{-1}(int(E))$.

(ii) implies (iii) is obvious by taking complement in (ii).

Suppose (iii) holds: Let $D$ be a BPFCS in $(Y, \sigma_p)$. Then $cl(D) = D$ and $\kappa^{-1}(D)$ is a BPFS in $(X, \tau_p)$. Now $\kappa^{-1}(D) = \kappa^{-1}(cl(D)) \subseteq int(\kappa^{-1}(D)) \subseteq \kappa^{-1}(D)$, by hypothesis. This implies $\kappa^{-1}(D)$ is a BPFOS in $(X, \tau_p)$ and hence $\kappa$ is a BPF$C$ continuous mapping. Since every BPF$C$ continuous mapping is BPF$C$RG continuous mapping, $\kappa$ is a BPF$C$RG continuous mapping.

**Theorem 3.24:** A BPF continuous mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ is a BPF$C$RG continuous mapping, if BPFRGO(X) = BPFRGC(X).
**Proof:** Let $D \subseteq Y$ be a BPFOS. By hypothesis, $\kappa^{-1}(D)$ is a BPFOS in $(X, \tau_p)$ and hence $\kappa^{-1}(D)$ is a BPFRGOS in $(X, \tau_p)$, as BPFRGO(X) = BPFRGC(X). Therefore $\kappa$ is BPF$C$RG continuous mapping.

**Theorem 3.25:** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a bijective mapping. Then $\kappa$ is a BPF$C$RG continuous mapping if $cl(\kappa(D)) \subseteq \kappa(int(D))$ for every BPFS $D$ in $(X, \tau_p)$.
**Proof:** Let $D$ be a BPFCS in $(Y, \sigma_p)$. Then $cl(D) = D$ and $\kappa^{-1}(D)$ is a BPFS in $(X, \tau_p)$. By hypothesis, $cl(\kappa(\kappa^{-1}(D))) \subseteq \kappa(int((\kappa^{-1}(D)))$ . Since $\kappa$ is a surjective function, $\kappa^{-1}(D)) = D$ . Therefore $D = cl(D) = cl(\kappa(\kappa^{-1}(D))) \subseteq \kappa(int(\kappa^{-1}(D)))$ . Now $\kappa^{-1}(D) \subseteq \kappa^{-1}(\kappa(int(\kappa^{-1}(D)))) = int(\kappa^{-1}(D)) \subseteq \kappa^{-1}(D)$. Hence $\kappa^{-1}(D)$ is a BPFOS in $(X, \tau_p)$ and a BPFRGOS in $(X, \tau_p)$. Thus $\kappa$ is a BPF$C$RG continuous mapping.

**Theorem 3.26:** If $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF$C$RG continuous mapping, then
(i) $\kappa$ is a BPF$C$ continuous mapping if $X$ is a $BPFR_cT_{1/2}$ space.
(ii) $\kappa$ is a BPF$C\alpha$ continuous mapping if X is a $BPFR_\alpha T_{1/2}$ space.
**Proof:** (i) Let $D$ be a BPFOS in $(Y, \sigma_p)$. Then $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$, by hypothesis. Since $(X, \tau_p)$ is a $BPFR_cT_{1/2}$ space, $\kappa^{-1}(D)$ is a BPFCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$C$ continuous mapping.

(ii) Let $D$ be a BPFOS in $(Y, \sigma_p)$. Then $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$, by hypothesis. Since $(X, \tau_p)$ is a $BPFR_\alpha T_{1/2}$ space, $\kappa^{-1}(D)$ is a BPFCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$C\alpha$ continuous mapping.

# 4. BIPOLAR PYTHAGOREAN FUZZY ALMOST REGULAR GENERALIZED CONTINUOUS MAPPINGS

In this section we are going to study the new concept of almost regular generalized continuous mapping in Bipolar Pythagorean Fuzzy Topological spaces.

**Definition 4.1:** A mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ is said to be a Bipolar Pythagorean Fuzzy Almost Regular Generalized Continuous mapping (BPF$a$RG continuous mapping) if $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$ for every BPFRCS $D$ in $(Y, \sigma_p)$.
**Example 4.2:** Let X={a,b}, Y={u,v}, $T_1$=(x, (0.6, 0.8), (0.5, 0.3), (-0.5, -0.7), (-0.4, -0.2)), $T_2$=(x, (0.3, 0.3), (0.8, 0.8), (-0.2, -0.2), (-0.7, -0.7)), $T_3$=(y, (0.7, 0.7), (0.4, 0.3), (-0.6, -0.6), (-0.3, -0.2)) and $T_4$=(Y, (0.3, 0.3), (0.9, 0.9), (-0.2, -0.2), (-0.8, -0.8)). Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on $X$ and $Y$ respectively. Define a mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ by $\kappa(a) = u$ and $\kappa(b) = v$. Here $\kappa$ is said to be BPF$a$RG continuous in $(X, \tau_p)$, as $\kappa^{-1}(T_3^c)$ = (x, (0.4, 0.3), (0.7, 0.7), (-0.3, -0.2), (-0.6, -0.6)) is BPFRG continuous in $(X, \tau_p)$ and $T_1$ is BPFROS in $(X, \tau_p)$ for every

$T_3^c = (y, (0.4,0.3), (0.7,0.7), (-0.3,-0.2), (-0.6,-0.6))$ is BPFRCS in $(Y, \sigma_p)$, as $cl(int(T_3^c)) = T_3^c$. Hence $\kappa$ is a BPF$a$RG continuous mapping.

**Theorem 4.3:** Every BPF continuous mapping is a BPF$a$RG continuous mapping but not conversely.

**Proof :** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF continuous mapping in $(Y, \sigma_p)$. Let $D$ be a BPFRCS in $(Y, \sigma_p)$. Since every BPFRCS is a BPFCS, $D$ is a BPFCS in $(Y, \sigma_p)$. Then, $\kappa^{-1}(D)$ is a BPFCS in $(X, \tau_p)$, by hypothesis. Since every BPFCS is a BPFRGCS, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$a$RG Continuous mapping.

**Example 4.4:** In Example 4.2, $\kappa^{-1}(T_3^c)$ = (x, (0.4, 0.3), (0.7, 0.7), (-0.3, -0.2), (-0.6, -0.6)) is BPFRGCS in $(X, \tau_p)$ for every BPFRCS in $(Y, \sigma_p)$ but not BPFCS in $(X, \tau_p)$ for every BPFRCS in $(Y, \sigma_p)$. Therefore $\kappa$ is a BPF$a$RG continuous mapping but not a BPF continuous mapping, since $cl(\kappa^{-1}(T_3^c)) = T_1^c \neq \kappa^{-1}(T_3^c)$.

**Theorem 4.5:** Every BPFR continuous mapping is a BPF$a$RG continuous mapping but not conversely.

**Proof :** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF continuous mapping in $(Y, \sigma_p)$. Let $D$ be a BPFRCS in $(Y, \sigma_p)$. Since every BPFRCS is a BPRGCS, $D$ is a BPFRGS in $(Y, \sigma_p)$. Then, $\kappa^{-1}(D)$ is a BPFRGS in $(X, \tau_p)$, by hypothesis. Hence $\kappa$ is a BPF$a$RG Continuous mapping.

**Example 4.6:** In Example 4.2, $\kappa^{-1}(T_3^c)$ = (x, (0.4, 0.3), (0.7, 0.7), (-0.3, -0.2), (-0.6, -0.6)) is BPFRGCS in $(X, \tau_p)$ for every BPFRCS in $(Y, \sigma_p)$ but not BPFRCS in $(X, \tau_p)$ for every BPFRCS in $(Y, \sigma_p)$. Therefore $\kappa$ is a BPF$a$RG continuous mapping but not a BPFR continuous mapping, since $int(cl(\kappa^{-1}(T_3^c))) = T_2 \neq \kappa^{-1}(T_3^c)$.

**Theorem 4.7:** Every BPF$\alpha$ continuous mapping is a BPF$a$RG continuous mapping but not conversely.

**Proof :** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF$\alpha$ continuous mapping. Let $D$ be a BPFRCS in $(Y, \sigma_p)$. Since every BPFRCS is a BPFCS, $D$ is a BPFCS in $(Y, \sigma_p)$. $\kappa^{-1}(D)$ is a BPF$\alpha$CS in $(X, \tau_p)$, by hypothesis. Since every BPF$\alpha$CS is a BPFRGCS, $f^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$a$RG continuous mapping.

**Example 4.8:** In Example 4.2, $\kappa^{-1}(T_3^c)$ = (x, (0.4, 0.3), (0.7, 0.7), (-0.3, -0.2), (-0.6, -0.6)) is BPFRGCS in $(X, \tau_p)$ for every BPFRCS in $(Y, \sigma_p)$ but not BPF$\alpha$CS in $(X, \tau_p)$ for every BPFRCS in $(Y, \sigma_p)$. Therefore $\kappa$ is a BPF$a$RG continuous mapping but not a BPF$\alpha$ continuous mapping, since $cl(int(cl(\kappa^{-1}(T_3^c)))) = T_1^c \neq \kappa^{-1}(T_3^c)$.

**Theorem 4.9:** Every BPF$\alpha$G continuous mapping is a BPF$a$RG continuous mapping but not conversely.

**Proof:** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPF$\alpha$G continuous mapping. Let $D$ be a BPFRCS in $(Y, \sigma_p)$. Since every BPFRCS is a BPF$\alpha$G, $D$ is a BPF$\alpha$GCS in $(Y, \sigma_p)$. $\kappa^{-1}(D)$ is a BPF$\alpha$GCS in $(X, \tau_p)$, by hypothesis. Since every BPF$\alpha$GCS is a BPFRGCS, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$a$RG continuous mapping.

**Example 4.10:** In Example 4.2, $\kappa^{-1}(T_3^c)$ = (x, (0.4, 0.3), (0.7, 0.7), (-0.3, -0.2), (-0.6, -0.6)) is BPFRGCS in $(X, \tau_p)$ for every BPFRCS in $(Y, \sigma_p)$ but not BPF$\alpha$GCS in $(X, \tau_p)$ for every BPFRCS in $(Y, \sigma_p)$. Therefore $\kappa$ is a BPF$a$RG continuous mapping but not a BPF$\alpha$G continuous mapping.

**Theorem 4.11:** Every BPFG continuous mapping is a BPF$a$RG continuous mapping but not conversely.

**Proof :** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFG continuous mapping. Let $D$ be a BPFRCS in $(Y, \sigma_p)$. Since every BPFRCS is a BPFGCS, $D$ is a BPFGCS in $(Y, \sigma_p)$. $\kappa^{-1}(D)$ is a BPFGCS in $(X, \tau_p)$, by hypothesis. Since every BPFGCS is a BPFRGCS, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$a$RG continuous mapping.

**Example 4.12:** In Example 4.2, $\kappa^{-1}(T_3^c)$ = (x, (0.4, 0.3), (0.7, 0.7), (-0.3, -0.2), (-0.6, -0.6)) is BPFRGCS in $(X, \tau_p)$ for every BPFRCS in $(Y, \sigma_p)$ but not BPFCS in $(X, \tau_p)$ for every BPFRCS in $(Y, \sigma_p)$. Therefore $\kappa$ is a BPF$a$RG continuous mapping but not a BPF$a$ continuous mapping, since $cl(\kappa^{-1}(T_3^c)) = T_1^c \neq \kappa^{-1}(T_3^c)$. Hence $\kappa$ is a BPF$a$RG continuous mapping.

**Remark 4.13:** BPFP continuous mapping and BPF $a$ RG continuous mappings are independent of eachother.

**Example 4.14:** Let X={a,b}, Y={u,v}, $T_1$=(x, (0.6, 0.4), (0.1, 0.1), (-0.7, -0.5), (-0.2, -0.2)), $T_2$=(x, (0, 0.1), (0.5, 0.5), (-0.1, -0.2), (-0.6, -0.6)), $T_3$=(y, (0.5, 0.6), (0, 0.1), (-0.6, -0.7), (-0.1, -0.2)) and $T_4$=(Y, (0, 0), (0.8, 0.8), (0, 0), (-0.7, -0.7)). Then $\tau_p$={$0_p, T_1, T_2, 1_p$} and $\sigma_p$ = {$0_p, T_3, T_4, 1_p$} are BPFTs on $X$ and $Y$ respectively. Define a mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ by $\kappa(a) = u$ and $\kappa(b) = v$ . The BPFS $T_3^c = (y, (0.4,0.3), (0.7,0.7), (-0.3, -0.2), (-0.6, -0.6))$ is BPFRCS in $(Y, \sigma_p)$. Then $\kappa^{-1}(T_3^c)$ = (x, (0, 0.1), (0.5, 0.6), (-0.6, -0.7), (-0.1, -0.2)) is a BPFS in $(X, \tau_p)$ and $\kappa^{-1}(T_3^c) \subseteq T_1$ where $T_1$ is a BPFROS in $(X, \tau_p)$. Now, $cl(int(\kappa^{-1}(T_3^c))) = 0_p \subseteq \kappa^{-1}(T_3^c)$. Here $\kappa^{-1}(T_3^c)$ is a BPFPCS in $(X, \tau_p)$ but not a BPFRGCS in $(X, \tau_p)$, since $cl(\kappa^{-1}(T_3^c)) = T_2^c \not\subseteq T_1$ but $\kappa^{-1}(T_3^c) \subseteq T_1$ where $T_1$ is a BPFROS in $(X, \tau_p)$. Therefore $\kappa$ is a BPFP continuous mapping but not a BPF$a$RG continuous mapping.

**Example 4.15:** Let X={a,b}, Y={u,v}, $T_1$=(x, (0.6, 0.5), (0.5, 0.4), (-0.6, -0.6), (-0.5, -0.5)), $T_2$=(y, (0.2, 0.3), (0.8, 0.9), (-0.1, -0.2), (-0.8, -0.9)), $T_3$=(y, (0.5, 0.6), (0.3, 0.4), (-0.6, -0.6), (-0.3, -0.3)) and $T_4$=(Y, (0.2, 0.1), (0.8, 0.9), (-0.1, -0.1), (-0.8, -0.8)). Then $\tau_p$={$0_p, T_1, T_2, 1_p$} and $\sigma_p$={$0_p, T_3, T_4, 1_p$} are BPFTs on $X$ and $Y$ respectively. Define a mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ by $\kappa(a) = u$ and $\kappa(b) = v$ . The BPFS $T_3^c = (y, (0.3,0.4), (0.5,0.6), (-0.3, -0.3), (-0.6, -0.6))$ is a BPFRCS in $(Y, \sigma_p)$ and $\kappa^{-1}(T_3^c)$ = (x, (0.3, 0.4), (0.5, 0.6), (-0.3, -0.3), (-0.6, -0.6)) is a BPFS in $(X, \tau_p)$. Now $cl(\kappa^{-1}(T_3^c)) = T_1^c \subseteq T_1$ and $\kappa^{-1}(T_3^c) \subseteq T_1$. Here $\kappa^{-1}(T_3^c)$ is a BPFRGCS in $(X, \tau_p)$ but not a BPFPCS in $(X, \tau_p)$, since $cl(int(\kappa^{-1}(T_3^c))) = T_2^c \not\subseteq \kappa^{-1}(T_3^c)$. Therefore $\kappa$ is a BPF $a$ RG continuous mapping but not a BPFP continuous mapping.

**Remark 4.16:** BPFS continuous mapping and BPF $a$ RG continuous mappings are independent of eachother.

**Example 4.17:** Let X={a,b}, Y={u,v}, $T_1$=(x, (0.5, 0.6), (0.4, 0.3), (-0.5, -0.5), (-0.3, -0.3)), $T_2$=(y, (0.2, 0.3), (0.6, 0.7), (-0.1, -0.2), (-0.6, -0.6)), $T_3$=(y, (0.5, 0.6), (0.4, 0.4), (-0.5, -0.5), (-0.3, -0.3)) and $T_4$=(Y, (0.2, 0.2), (0.8, 0.9), (-0.1, -0.1),( -0.8, -0.8)), Then $\tau_p$={$0_p, T_1, T_2, 1_p$} and $\sigma_p$={$0_p, T_3, T_4, 1_p$} are BPFTs on $X$ and $Y$ respectively. Define a mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ by $\kappa(a) = u$ and $\kappa(b) = v$. The BPFS $T_3^c$ = (y, (0.4, 0.4), (0.5, 0.6), (-0.3, -0.2), (-0.5, -0.5)) is a BPFRCS in $(Y, \sigma_p)$. $\kappa^{-1}(T_3^c)$ = (x, (0.4, 0.4), (0.5, 0.6), (-0.3, -0.2), (-0.5, -0.5)) is a BPFS in $(X, \tau_p)$. Now $cl(\kappa^{-1}(T_3^c)) = T_1^c \subseteq T_1$. Therefore $\kappa^{-1}(T_3^c)$ is a BPFRGCS in $(X, \tau_p)$ but not a BPFSCS in $(X, \tau_p)$, since $T_3^c$ is a BPFCS in $(Y, \sigma_p)$ but $int(cl(\kappa^{-1}(T_3^c))) = int(T_2^c) = T_1 \subseteq \kappa^{-1}(T_3^c)$. Here $\kappa^{-1}(T_3^c)$ is a BPFSCS in $(X, \tau_p)$ but not a BPFRGCS in $(X, \tau_p)$ , since $cl(\kappa^{-1}(T_3^c)) = T_1^c \not\subseteq T_1$ where $T_1$ is a BPFROS in $(X, \tau_p)$. Therefore $\kappa$ is a BPFS continuous mapping but not a BPF$a$RG continuous mapping.

**Example 4.18:** Let X={a,b}, Y={u,v}, $T_1$=(x, (0.6, 0.7), (0.5, 0.3), (-0.5, -0.6), (-0.4, -0.2)), $T_2$=(x, (0.3, 0.3), (0.8, 0.8), (-0.2, -0.2), (-0.7, -0.7)), $T_3$=(y, (0.7, 0.7), (0.4, 0.3), (-0.6, -0.6), (-0.3, -0.2)) and $T_4$=(Y, (0.3, 0.3), (0.9, 0.9), (-0.2, -0.2), (-0.8, -0.8)). Then $\tau_p$={$0_p, T_1, T_2, 1_p$} and $\sigma_p$={$0_p, T_3, T_4, 1_p$} are BPFTs on $X$ and $Y$ respectively. Define a mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ by $\kappa(a) = u$ and $\kappa(b) = v$ . The BPFS $T_3^c = (y, (0.4,0.3), (0.7,0.7), (-0.3, -0.2), (-0.6, -0.6))$ is BPFRCS in $(Y, \sigma_p)$. Then $\kappa^{-1}(T_3^c)$ = (x, (0.4, 0.3), (0.7, 0.7), (-0.3, -0.2), (-0.6, -0.6)) is a BPFS in $(X, \tau_p)$. Then $\kappa^{-1}(T_3^c) \subseteq T_1$ where $T_1$ is a BPFROS in $(X, \tau_p)$. Now, $int(cl(\kappa^{-1}(T_3^c))) = int(T_1^c) = T_1 \not\subseteq \kappa^{-1}(T_3^c)$. Here $\kappa^{-1}(T_3^c)$ is not a BPFSCS in $(X, \tau_p)$ but a BPFRGCS in $(X, \tau_p)$, since $cl(\kappa^{-1}(T_3^c)) = T_1^c \subseteq T_1$

but $\kappa^{-1}(T_3^c) \subseteq T_1$ where $T_1$ is a BPFROS in $(X, \tau_p)$. Therefore $\kappa$ is not a BPFS continuous mapping but a BPF$a$RG continuous mapping.

**Theorem 4.19:** A mapping $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ is a BPF$a$RG continuous mapping iff the inverse image of each BPFROS in $(Y, \sigma_p)$ is a BPFRGOS in X.
**Proof: Necessity:** Let $D$ be a BPFROS in $(Y, \sigma_p)$. This implies $D^c$ is a BPFRCS in $(Y, \sigma_p)$. Since $\kappa$ is an BPF$a$RG continuous mapping, $\kappa^{-1}(D^c)$ is a BPFRGCS in $(X, \tau_p)$. Since $\kappa^{-1}(D^c) = (\kappa^{-1}(D))^c$, $\kappa^{-1}(D)$ is a BPFRGOS in $(X, \tau_p)$.
**Sufficiency:** Let $D$ be a BPFRCS in $(Y, \sigma_p)$. This implies $D^c$ is a BPFROS in $(Y, \sigma_p)$. By hypothesis, $\kappa^{-1}(D^c)$ is a BPFRGOS in $(X, \tau_p)$. Since $\kappa^{-1}(D^c) = (\kappa^{-1}(D))^c$, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$a$RG continuous mapping.

**Theorem 4.20:** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping where $\kappa^{-1}(D)$ is a BPFRCS in $(X, \tau_p)$ for every BPFCS in $(Y, \sigma_p)$. Then $\kappa$ is a BPF$a$RG continuous mapping but not conversely.
**Proof:** Let $D$ be a BPFRCS in $(Y, \sigma_p)$. Since every BPFRCS is a BPFCS, $D$ is a BPFCS in $(Y, \sigma_p)$. Then $\kappa^{-1}(D)$ is a BPFRCS in $(X, \tau_p)$. Since every BPFRCS is an BPFRGCS, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\kappa$ is a BPF$a$RG continuous mapping.

**Theorem 4.21:** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping. If $\kappa^{-1}(int(D)) \subseteq int(\kappa^{-1}(D))$ for every BPFS B in $(Y, \sigma_p)$, then $\kappa$ is a BPF$a$RG continuous mapping.
**Proof:** Let $D$ be a BPFROS in $(Y, \sigma_p)$. By hypothesis, $\kappa^{-1}(int(D)) \subseteq int(\kappa^{-1}(D))$. Since $D$ is a BPFROS, it is a BPFOS in $(Y, \sigma_p)$. Therefore $int(D) = D$. Hence $\kappa^{-1}(D) = \kappa^{-1}(int(D)) \subseteq int(\kappa^{-1}(D)) \subseteq \kappa^{-1}(D)$. Therefore $\kappa^{-1}(D) = int(\kappa^{-1}(D))$. This implies $\kappa^{-1}(D)$ is a BPFOS in $(X, \tau_p)$ and hence $\kappa^{-1}(D)$ is a BPFRGOS in $(X, \tau_p)$. Thus $\kappa$ is a BPF$a$RG continuous mapping.

**Remark 4.22:** The converse of the above theorem 4.21 is true if $D$ is a BPFROS in $(Y, \sigma_p)$ and $(X, \tau_p)$ is a $BPFRT_{1/2}$ space.
**Proof:** Let $\kappa$ be a BPF$a$RG continuous mapping and $D$ be a BPFROS in $(Y, \sigma_p)$. Then $\kappa^{-1}(D)$ is a BPFRGOS in $(X, \tau_p)$. Since $(X, \tau_p)$ is a $BPFRT_{1/2}$ space, $\kappa^{-1}(D)$ is a BPFOS in $(X, \tau_p)$. This implies $\kappa^{-1}(D) = int(\kappa^{-1}(D))$. Now $\kappa^{-1}(int(D)) \subseteq \kappa^{-1}(D) = int(\kappa^{-1}(D))$. Therefore $\kappa^{-1}(int(D)) \subseteq int(\kappa^{-1}(D))$.

**Theorem 4.23:** Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping. If $cl(\kappa^{-1}(D)) \subseteq \kappa^{-1}(cl(D))$ for every BPFS $D$ in $(Y, \sigma_p)$, then $\kappa$ is a BPF$a$RG continuous mapping.
**Proof:** Let $D$ be a BPFRCS in $(Y, \sigma_p)$. By hypothesis, $cl(\kappa^{-1}(D)) \subseteq \kappa^{-1}(cl(D))$. Since $D$ is a BPFRCS, it is a BPFCS in $(Y, \sigma_p)$. Therefore $cl(D) = D$. Hence $\kappa^{-1}(D) = \kappa^{-1}(cl(D)) \supseteq cl(\kappa^{-1}(D)) \supseteq \kappa^{-1}(D)$. Therefore $\kappa^{-1}(D) = cl(\kappa^{-1}(D))$. This implies $\kappa^{-1}(D)$ is a BPFCS in $(X, \tau_p)$ and hence $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Thus $\kappa$ is a BPF$a$RG continuous mapping.

**Theorem 4.24:**: Let $\kappa: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping where $(X, \tau_p)$ is a $BPFRT_{1/2}$ space. If $\kappa$ is a BPF$a$RG continuous mapping, then $cl(\kappa^{-1}(D)) \subseteq \kappa^{-1}(cl(D)$ for every BPFRCS $D$ in $(Y, \sigma_p)$.
**Proof:** Let $D$ be a BPFRCS in $(Y, \sigma_p)$. By hypothesis, $\kappa^{-1}(D)$ is a BPFRGCS in $(X, \tau_p)$. Since $(X, \tau_p)$ is a $BPFRT_{1/2}$ space, $\kappa^{-1}(D)$ is a BPFCS in $(X, \tau_p)$.
This implies $cl(\kappa^{-1}(D)) = \kappa^{-1}(D)$.
Now $cl(\kappa^{-1}(D)) \subseteq \kappa^{-1}(D) \cup cl(\kappa^{-1}(D)) \subseteq cl(\kappa^{-1}(D)) = \kappa^{-1}(D) = \kappa^{-1}(cl(D))$, as every BPFRCS is a BPFCS. Hence $cl(\kappa^{-1}(D)) \subseteq \kappa^{-1}(cl(D))$.

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