Heat Source Effect on MHD Fluid Flow Over a Moving Vertical Plate in the presence of Chemical Reaction with Convective surface Boundary Conditions

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Abstract: *This paper focuses on the effects of a convective boundary condition and internal heat source along with chemical reaction on the laminar magneto hydrodynamic (MHD) flow involving a mixed convection heat and mass transfer flow over a vertically moving plate. The governing equations for such flows are drawn and solved using the Keller box techniques on reduction to set of ordinary differential equations. Graphical representations are made use of to understand the outcomes of physical parameters on the concentration, temperature and velocity profiles. A tabular depiction is used for the values of skin friction, Heat and Mass transfer coefficients around the plate.*

Keywords: **MHD, Mixed Convection, Keller Box Techniques, Heat and Mass transfer.**

1. INTRODUCTION

Many industrial processes have applications of convective flows involving mass and heat transfer influenced by magnetic field with heat source and chemical reaction. Practical applications of this process are predominantly found in petroleum industry, chemical industry and in cooling of nuclear reactors. Convection flows occur naturally due to differences in concentration and temperature along with other combined effects. Rate of heat transfer may be qualitatively varied due to a difference in concentration. Combustion modelling can be improved by the field of heat propagation in many areas due to the chemical reactions of both exothermic and endothermic type and natural convection with heat generation. In this way Chien –Hsin Chen and Cha-Kuang Chen [1] studied Non-Darcian mixed convection along a vertical plate embedded in a porous medium. Unsteady laminar free convection from a vertical cone with uniform surface heat flux was studied by Bapuji et.al [2]. R. Muthukumaraswamy and P. Ganesan [3] investigated an unsteady flow past an impulsively started vertical plate with heat and mass transfer. S.Mukhopadyay and G.C Layek [4] presented the radiation effect on forced convective flow and heat transfer over a porous plate in a porous medium. Mohamed E.Ali [5] investigated the effect of lateral mass flux on the natural convection boundary layers induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation. Dulal Pal. Hiranmoy modal [6] presented the radiation effects on combined convection over a vertical flat embedded in a porous medium of variable porosity. Anjali Devi and Kandaswamy [7] have presented the chemical reaction effects on heat amd mass transfer flow along a semi – infinite horizontal plate. A.R Vijayalakshmi [8] studied Radiation effects on free- convection flow past an impulsively started vertical plate in rotating fluid. A.A.Babar [9] considered the effects of chemical reaction on MHD free convection and mass transfer flow of a micro polar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. Shankar Goud [10] et.al investigated on stagnation point flow through a porous medium towards stretching surface in the presence of heat generation. Rajeswari et.al [11] studied

chemical reaction, heat and mass transfer on nonlinear MHD boundary layer floe through a vertical porous surface in presence of suction. Effect of chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface embedded in a porous media considering Soret an Dufour number by M.A Mansour et.al [12]. Mohammad Mehdi rashidi et.al [13] analyzed free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects. B.ShankarGoud [14] studied on MHD flow past a vertical oscillating plate with radiation andchemical reaction in porous medium- finite difference method. K.Javaherdeh et.al [15] investigated natural convection heat and mass transfer in MHD fluid flow past a moving vertical plate with variable surface temperature and concentration in porous medium. O.D.Makinde [16] has studied On MHD heat and mass transfer over a moving vertical plate with a convective surface boundary condition.

F.S.Ibrahim *et.al* [17] have analyzed "studied effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction. H.S Takhar et.al [18] have studied the unsteady mixed convection flow from a rotating vertical cone with a magnetic field. Gnaneshwara reddy [19] has analyzed influence of thermal radiation, viscous dissipation and Hall current on MHD convection flow over a stretched vertical flat plate. O. Anwar Beg et.al [20] discussed numerical study of free convection magnetohydrodynamic heat and mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects.

This paper deals with the heat source effect on MHD fluid flow over a moving vertical plate in the presence of chemical reaction with convective surface boundary conditions and set of ODE are solved by Keller box method. Final results are presented through graphs.

2. MATHEMATICAL FORMULATION

In this problem we consider a steady two-dimensional incompressible flow caused by moving vertical plate which is placed electrically taking fluid over a moving perpendicular flat plate at a temperature T_∞ taking heat source & chemical reaction into account.

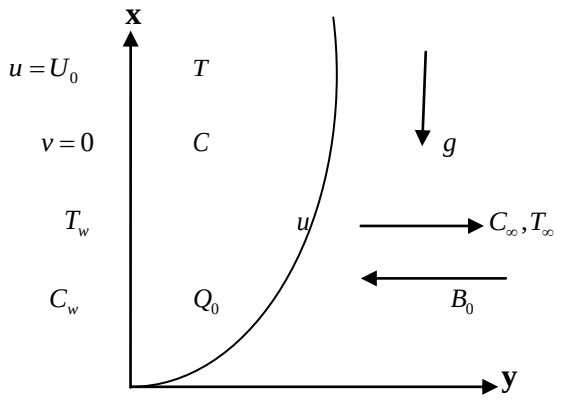

Fig. 1. Flow geometry

Here the $x-$axis is taken along the plate and y-axis is perpendicular to it. The fluid properties are assumed to be constant and magnetic field is applied in the negative direction of $y-$ axis. The governing equations are identifying the flow under the Bouessinesq approximation can be written as

Eqn. of continuity: $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ --- (1)

Eqn. of momentum: $u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=v\frac{\partial^2 v}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho}+g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right)$ --- (2)

Eqn. of energy: $u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}\left(T-T_\infty\right)$ --- (3)

Concentration Equation: $u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 v}{\partial y^2}-Kr'C$ --- (4)

Where u, v are the fluid velocity components in the x, y direction. Here T and C are the temperature and concentration variables, B_0 the strength of magnetic field, v is the kinematic viscosity, α is the thermal diffusivity, β,β^* are the thermal expansion coefficient, and solutal expansion coefficient, ρ,g,σ and Q_0 the fluid density, the gravitational acceleration, the electrical conductivity and the heat source, C_p is the specific heat at constant pressure and Kr' is the chemical reaction rate on the species concentration. In the above equations, various assumptions have been made. The plate is non-conducting, and the effects of viscous dissipation, heating, Hall effects and induced fields are neglected. The physical properties that are heat capacity, viscosity, mass diffusivity, and the thermal diffusivity of the fluid remain constant throughout the fluid.

The appropriate boundary conditions are for the velocity, temperature and concentrations are

$$\left.\begin{array}{l} y=0: \quad u(x,0)=U_0, v(x,0)=0, -k\frac{\partial T}{\partial y}(x,0)=h_w\left[T_w-T(x,0)\right], \\ y\to\infty: \quad C_w=(x,0)=Ax^\lambda+C_\infty, u(x,\infty)=0, T(x,0)=T_\infty, C(x,\infty)=C_\infty \end{array}\right\} \quad \text{--- (5)}$$

Where C_w the species concentration at the plate surface, A is the constant. λ is the power index of concentration, U_0,k and C_∞ are the plate velocity and the thermal conductivity coefficient and the concentration of the fluid away from the plate. The non - linear partial differential equation can be reduced into system of ordinary differential equation by the following similarity transformations can be introduced:

$$\begin{array}{l} u=U_0 f'(\eta), v=-\frac{1}{2}\sqrt{\frac{vU_0}{x}}f(\eta)+\frac{U_0 y}{2x}f'(\eta), \\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \eta=y\sqrt{\frac{U_0}{vx}} \end{array} \quad \text{--- (6)}$$

Substituting eq. (6) into Eqs. (1) - (5) we get the following equations and boundary conditions:

$$f'''+\frac{1}{2}ff''-H_a f'+Gr\theta+Gc\phi=0 \quad \text{--- (7)}$$

$$\theta''+\frac{1}{2}\Pr f\theta'+\Pr S\theta=0 \quad \text{--- (8)}$$

$$\phi''+\frac{1}{2}Scf\phi'-ScKr\left(\phi+N\right)=0 \quad \text{--- (9)}$$

Boundary conditions:

$$\left.\begin{array}{l} f(0)=0, f'(0)=1, \theta'(0)=Bi\left[\theta(0)-1\right], \phi(0)=1 \\ f'(\infty)=, \theta(\infty)=0, \phi(\infty)=1 \end{array}\right\} \quad \text{--- (10)}$$

Where prime denotes the differentiation with respect the variable η and the dimensionless parameters are defined:

$$Ha=\frac{\sigma B_0^2 x}{\rho U_0}, Gr=\frac{g\beta\left(T_w-T_\infty\right)x}{U_0^2}, Gc=\frac{g\beta^*\left(C_w-C_\infty\right)x}{U_0^2}, \Pr=\frac{\nu}{\alpha},$$
$$Bi=\frac{h_w}{k}\sqrt{\frac{\nu x}{U_0}}, Sc=\frac{\nu}{D}, S=\frac{Q_0 x}{U_0\rho C_p}, Kr=\frac{Kr' x}{U_0}, N=\frac{C_\infty}{C_w-C_\infty} \quad \text{--- (11)}$$

Here, Gr, Gc, Pr and Sc, and local thermal Grashof, modified Grashof, Prandtl numbers parameter, and Schmidt numbers, Ha, Bi, S, Kr and N are the local magnetic field ,local convective heat transfer, local heat source , local chemical reaction, and is the concentration difference parameters.

The local skin friction coefficient, the local Nusselt number, the local Sherwood number, and the plate surface temperature are calculated in terms of $f''(0), -\theta'(0)$, $-\phi'(0)$ and $\theta(0)$ respectively. It can be observed that the local parameters Ha, Gr, Gc, Bi, S and Kr in (7)-(8) are functions of x and give local similarity solution. In order to have a true similarity solution we assume the following relation.

$$h_w=\frac{a}{\sqrt{x}}, \sigma=\frac{b}{x}, \beta=\frac{c}{x}, \beta^*=\frac{d}{x}, Q_0=\frac{e}{x}, Kr'=\frac{m}{x} \quad \text{--- (12)}$$

Where a, b, c, d, e and m are the constants with appropriate dimensions. The parameters equations (11) are now independent of x.

3. NUMERICAL SOLUTION

The coupled non-linear ordinary differential equations (7) - (9) subject to the boundary conditions (10) are solved numerically by using implicit finite difference strategy known as Keller Box method. This method convergence is depends on by choosing initial guesses $f_0(\eta)=1-e^{-\eta}, \theta_0=\frac{Bi}{1+Bi}e^{-\eta}, \phi_0=e^{-\eta}$. In this method the non linear differential equations are changed into simultaneous equations of first order and further, transformed into initial value problem. A step size 0.01 is chosen to satisfy the convergence criterion of 10^{-5} in this study.

4. RESULTS AND DISCUSSIONS

A numerical solution of the system of ordinary differential equation problem with a boundary values was obtained by using implicit finite difference scheme i.e Keller Box method. There are varied applications to the problem. In order to gather the far field boundary conditions are endless, the computational domain is set sufficiently large. The results of $f''(0)$, $-\theta'(0), \theta(0)$ and $\phi'(0)$, obtained for varying, Gc, Bi, Gr, Ha, Sc and Pr values at $S=Kr=0$ are presented in Table 1. An influence of parameters, Pr, Sc and Kr and S on skin friction is observed in the current problem because of the momentum equation together with heat and mass transfer equation.

Table 1: Results of $f''(0)$, $-\theta'(0)$, $\theta(0)$ and $-\phi'(0)$ for varying values of Gc, Bi, Gr Ha, Pr, and Sc when $S=Kr=0$.

Table:1	Gr	Bi	Gc	Sc	Ha	Pr	$f''(0)$	$-\theta'(0)$	$\theta(0)$	$-\phi'(0)$
	0.1	1.0	0.1	0.62	0.1	0.72	−0.352136	0.273153	0.726846	0.341029
	0.5	0.1	0.1	0.62	0.1	0.72	−0.322212	0.079173	0.208264	0.345130
	0.1	0.1	0.1	0.62	0.1	0.72	−0.231251	0.079691	0.203088	0.356665
	0.1	0.1	0.1	0.62	0.1	0.72	−0.026410	0.080711	0.192889	0.381395
	0.1	0.1	0.1	0.62	0.1	0.72	0.379918	0.082040	0.179592	0.417669
	0.1	0.1	0.1	0.62	5.0	0.72	−2.217928	0.066156	0.338435	0.180664
	0.1	0.1	0.1	0.62	0.1	1.0	−0.407908	0.081935	0.180640	0.332518
	0.1	0.1	0.1	0.62	0.1	7.10	−0.421431	0.093348	0.066515	0.330843
	0.1	0.1	0.1	0.78	0.1	0.72	−0.411704	0.078484	0.215159	0.384455
	0.1	0.1	0.1	2.63	0.1	0.72	−0.453094	0.077915	0.220841	0.798146

Tables 2 and 3 describe the influence of chemical parameter and heat source on $f''(0)$, $-\theta'(0)$, $\theta(0)$ and $-\phi'(0)$ for various non dimensional flow parameters. The other parameters are $Gc = 0.1, Ha = 0.1, Gr = 0.1, Sc = 0.62$ and $N = 0.01$ .Table 2 depicts an increase of source parameter leading to an increase in both skin friction and Nusselt number. On the other hand, the plate surface number and Sherwood number tend to decrease in the same condition. Table 3: $Bi = Gr = Gc = 0.1, \Pr = 0.72$ and $N = 0.01$ are fixed. Computations of other parameter form different values are as follows.

Table:1	*Bi*	*Pr*	*S*	*Kr*	$f''(0)$	$-\theta'(0)$	$\theta(0)$	$-\phi'(0)$
	0.5	0.72	0.1	0.1	−0.358517	0.167824	0.664351	0.428429
	1.0	0.72	0.1	0.1	−0.344220	0.204287	0.795712	0.430013
	1.0	0.72	0.3	0.1	−0.410247	0.283270	0.716729	0.413375
	1.0	0.72	0.3	0.3	−0.419822	0.265179	0.734820	0.552735
	1.0	0.72	0.3	0.6	−0.428749	0.253741	0.746258	0.708364
	1.0	1.0	0.3	0.6	−0.412617	0.225282	0.774717	0.709739

Increased values of strength of chemically reacting substances indicate an increased local skin friction, plate surface temperature and Sherwood number along with decreased local Nusselt number.

Table :3	*Sc*	*S*	*Kr*	$f''(0)$	$-\theta'(0)$	$\theta(0)$	$-\phi'(0)$
	0.62	0.05	0.05	−0.604781	0.154830	0.548301	0.376062
	0.62	0.1	0.05	−0.358517	0.167824	0.664351	0.428429
	0.62	0.3	0.05	−0.344220	0.204287	0.795712	0.430013
	0.62	0.1	0.5	−0.410247	0.283270	0.716729	0.413375
	0.62	0.1	1.0	−0.419822	0.265179	0.734820	0.552735
	0.78	0.1	1.0	−0.428749	0.253741	0.746258	0.708364

It can be observed from table 2 that an increase in magnitude of the local skin friction and local Nusselt number is driven by the fluid flow Prandtl number. It is also clear that fluid flow Prandtl number decreases the value of plate surface temperature along with local Sherwood number. A lower convective resistance or outside resistance lowers the magnitude of local skin friction. Alternatively, the values of Nusselt number and plate surface temperature, local Sherwood number increase in the above scenario Table 3 depicts the increase in Schmidt number leads to an increase in magnitude of local skin friction and decreased Nusselt number and plate surface temperature.

Figures 2 to 7 describe velocity temperature and concentration, profiles with numerical values for several flow arguments involved in this problem. Figure 2 illustrates both the effects of magnetic parameter on the velocity field in the scenarios of presence and absence of source and chemical reaction parameter. The velocity profile diminishes with an increase in magnetic parameter without source and chemical reaction parameter. This phenomenon can be explained by using Lorentz force which acts against the fluid flow when a magnetic field is applied in a direction perpendicular to the flow. Figure 3 depicts a raise in the velocity summary near the boundary layer on an increase in convective heat parameter. This can be justified by the fluid closest to the right plane of the plate rises faster as it becomes lighter by hot fluid. Figure 4 describes higher values of temperature profile with an increased magnetic parameter. From this, it can be understood that the fluid gets heated with an applied magnetic field thereby reducing the heat transfer from the wall. Also an increase in heat source and chemical reaction parameter leads to increased temperature profile. Figure 5 illustrates an increased convective heat parameter in the plate surface steering an increase in boundary layer thickness. Figure 6 depicts the applied magnetic field inducing an increased concentration of boundary layer. Figure7

illustrate the decreased concentration profile with higher values of both chemical and heat source parameter.

5. CONCLUSIONS

The present numerical assessments have been completed for heat & mass transfer of MHD flow over a moving plate in existence of S and Kr alongside convective surface boundary condition. Keller Box technique has been utilized to understand the dimensionless velocity, heat & mass boundary layer conditions. It has been demonstrated that the magnitude of local skin friction and local Nusselt number increment while the plate surface temperature and Sherwood number decrease with an expansion in S. The expansion in the quality of compound responding substances causes an increment in the magnitude of the local skin friction, the plate temperature, and Sherwood number yet inverse conduct is examined for local Nusselt number. The velocity summery diminishes by expanding Ha and even the expansion is extra noticeable with an expansion in S and Kr parameter. The thermal boundary layer thickness increments with the expansion of S, plate surface convective S and Sc while the mass flux boundary layer thickness diminishes. Also, the thermal boundary layer thickness, the mass boundary layer, and velocity diminish as Pr value increments.

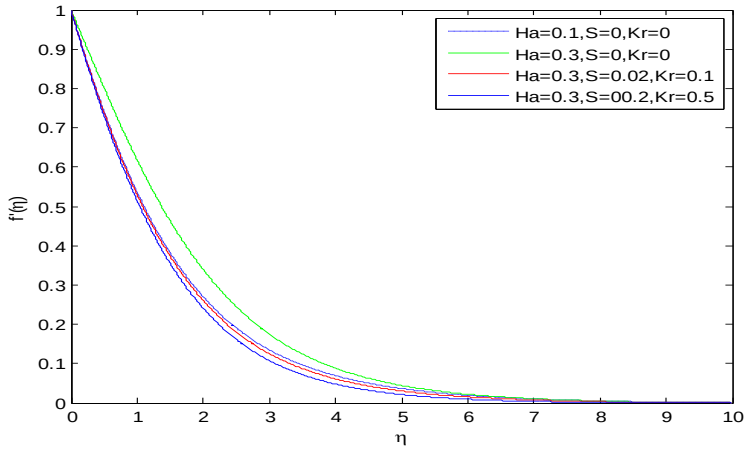

Fig.2. Velocity profile for altered values of Ha , S and Kr for $Gr = 0.1$, $Gc = 0.1$, $Sc = 0.62$, $Bi = 0.1$, $N = 0.01$ and $\Pr = 0.72$

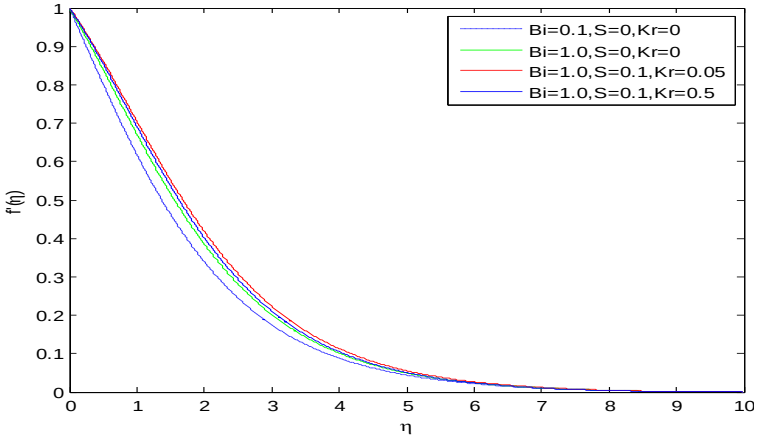

Fig.3. Velocity profile for changed Values of Bi , S and Kr for $Ha = 0.1$, $Gr = 0.1$, $Gc = 0.1$, $Sc = 0.62$, $N = 0.01$ and $\Pr = 0.72$

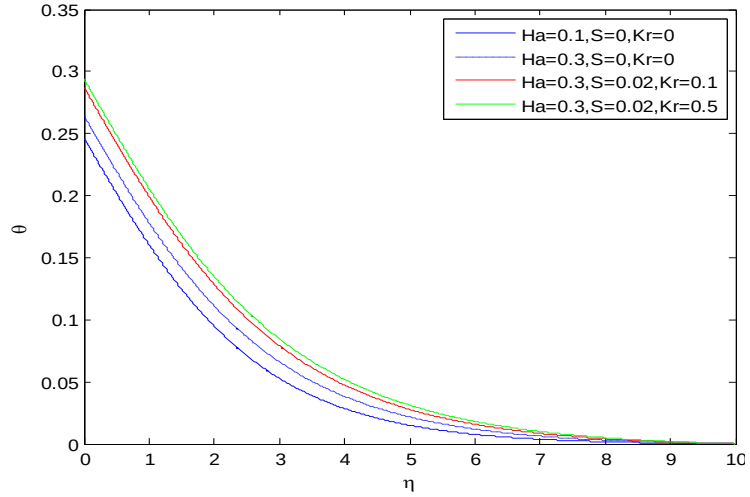

Fig.4. Temperature profiles for different values of Ha, S and Kr for $Gr = 0.1$, $Gc = 0.1$, $Bi = 0.1$, $P\mathrm{r} = 0.72$ and $N = 0.01$

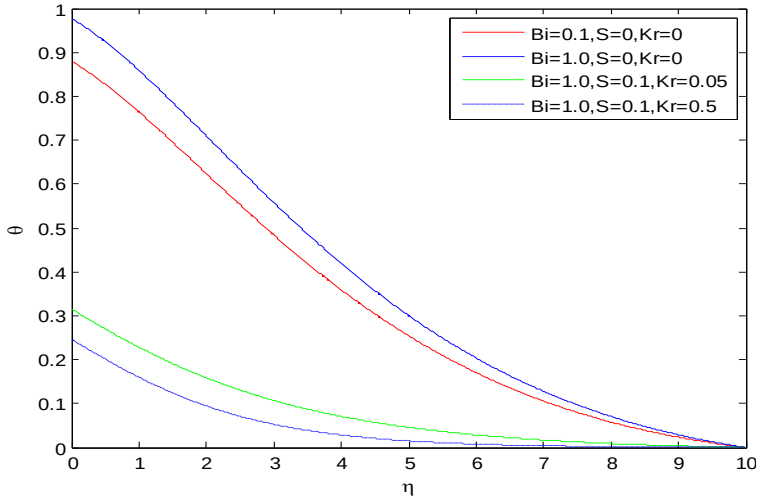

Fig. 5. Temperature profiles for changed values of Bi, S and, Kr for $Ha = 0.1$, $Gr = 0.1$, $Gc = 0.1$, $Sc = 0.62$, $Bi = 0.1$, $P\mathrm{r} = 0.72$ and $N = 0.01$

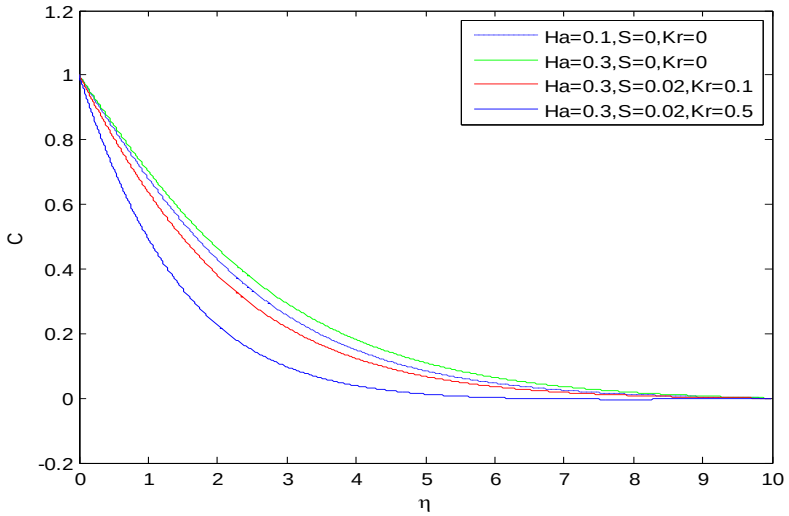

Fig. 6. Concentration profiles for various values Ha, S and Kr for $Gr = 0.1$, $Sc = 0.62$, $Bi = 0.1$, $P\mathrm{r} = 0.72$ and $N = 0.01$.

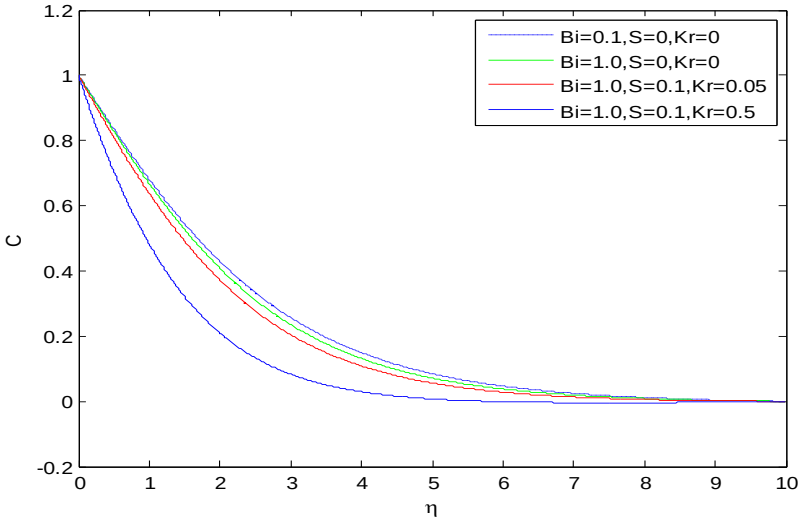

Fig.7. Concentration profiles for changed values Bi, S and Kr for $Ha = 0.1$, $Gr = 0.1$, $Sc = 0.62$, $P\mathrm{r} = 0.72$ and $N = 0.01$.

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