Analytical study on Biomedical and its Mechanisms using Casson Fluid over a vertical plate in presence of Heat Absorption

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Abstract: The present study aims to analyze the heat absorption effect on unsteady Casson fluid flow over a vertical permeable semi-infinite plate. The time-dependent wall suction is assumed to occur at the permeable surface. The non-Newtonian fluid behavior is characterized by using the Casson fluid model. Casson fluid model could be chosen for human blood flow investigations because human blood has many constituents like fibrinogen, protein, red blood cells, etc and also the blood flow needs to be checked (surgery) and the heat transfer rate must be controlled (therapy). The coupled non-linear ordinary differential equations (ODE's) are solved by perturbation technique. The impact of sundry parameters on the velocity and temperature are analyzed through graphs as well as the friction factor coefficient and the rates of heat transfer coefficient are computed.

Keywords: Casson fluid, Porosity, Heat Absorption, Unsteady state.

1. Introduction:

Different models of non-Newtonian fluids based on their diverse flow behaviors have been proposed by the researchers. The most commonly used power-law model has tendency to describe shear-thinning as well as shear-thickening behavior. However it cannot explain the normal stress differences in the flow. Maxwell model is considered to be the simplest viscoelastic models. This model is able to predict the effects of fluid relaxation time on its boundary layer. The Casson fluid model is considered to characterize the non-Newtonian behaviour of the blood. This phenomenon induced by contraction and relaxation of the waves propagating along the walls of tube/channel is recognized. The phenomenon exists in many organisms, in a different range of organs, in physiological and industrial processes. The mechanisms of transportation of urine from the kidney to the bladder through the ureter, movement of ova through the Fallopian tubes and the flow of blood with viscous motion in small blood vessels. The model is important in industrial pumping and medical devices e.g. transport of noxious fluids in the nuclear industry, roller pumping in the printing industry and the heart-

lung machine. The flow and heat transfer of blood have significant clinical importance during the stages where the blood flow needs to be checked (surgery) and the heat transfer rate must be controlled (therapy). In fact that Casson fluid is well suitable for blood flow model.

The study of non-Newtonian fluid flows plays a vital role in many branches of chemical and material processing engineering and exhibit shear-stress-strain relationships which diverge significantly from the Newtonian type fluid models. The different types of non-Newton fluids are power-law fluid flow, couple stress fluid, visco-elastic fluid, micropolar fluid, Casson fluid etc. In spite of these models, Casson fluid model is one of the most important models in non-Newtonian fluids. Physically, the Casson fluid is a plastic fluid that exhibits thinning characteristics. This model was useful for fluids containing rod like solids and is often applied to molten chocolate and blood. Sometimes the Casson fluid model is stated to fit rheological data better than general viscoplastic models for several materials [1-3]. Jelly, tomato sauce, honey, soup and concentrated fruit juices, etc are the examples of Casson fluids. Human blood can also be considered as Casson fluid. Due to the presence of several substances like, protein, fibrinogen and globulin in aqueous base plasma, human red blood cells can form a chainlike structure, known as aggregates or rouleaux. Charan Kumar et al. [4] have been reported that chemical reaction and soret effects on casson mhd fluid over a vertical plate. Vedavati et al. [5] have been explained the chemical reaction, radiation and dufour effects on casson magneto hydro dynamics fluid flow over a vertical plate with heat source /sink. Dharmaiah et al. [6] have been discussed the effect of chemical reaction on mhd casson fluid flow past an inclined surface with radiation. Sahin [7] studied the transient heat conduction over a semi-infinite solid in the presence of spatially decaying exponential heat generation. The free convection unsteady hydromagnetic flow over an inclined porous plate with a constant heat and mass fluxes in the presence of thermal radiation, heat source and chemical reaction was analyzed by Geethan Kumar et al. [8]. Animasaun et al. [9] studied the free convective hydromagnetic Casson fluid flow over an exponentially stretching surface with suction and found that the improving values of variable plastic dynamic viscosity parameter of Casson fluid corresponds to enhanced the fluid velocity and reduction of the temperature within the boundary layer. The free convection Casson fluid slip flow over an oscillating vertical plate with constant wall temperature is investigated by Imran et al. [10]. Hussanan et al. [11] considered the impact of Newtonian heating on Casson fluid flow past an oscillating vertical plate. Nadeem et al. [12] have discussed the hydromagnetic 3D Casson fluid flow past a porous linearly stretching sheet. Casson fluid flow in a pipe filled with homogeneous porous medium was reported by Dash et al. [13].

The present study aims to analyze the heat absorption effect on unsteady Casson fluid flow over a vertical permeable semi-infinite plate. The time-dependent wall suction is assumed to occur at the permeable surface. The non-Newtonian fluid behavior is characterized by using the Casson fluid model. The coupled non-linear ordinary differential equations (ODE's) are solved by perturbation technique. The impact of sundry parameters on the velocity and temperature are analyzed through graphs as well as the friction factor coefficient and the rate of heat coefficient are computed.

2. Problem Formulation:

Consider an unsteady Casson fluid flow past a semi-infinite vertical plate embedded in a porous medium.

- It is assumed that there is no applied voltage which implies the absence of an electrical field.
- It is assumed that all the thermo physical properties are to be constant of the linear momentum equation which is approximated according to the Boussinesq approximation.
- The plate moves with velocity u_p.

The rheological equation of state for an isotropic and viscous incompressible laminar Casson fluid flow is written as [14, 15].

$$\tau_{ij} = \begin{cases} \left(\mu_B + p_y / \sqrt{2\pi}\right) 2e_{ij}, \pi > \pi_c \\ \left(\mu_B + p_y / \sqrt{2\pi_c}\right) 2e_{ij}, \pi < \pi_c \end{cases}$$

Where μ_B is plastic dynamic viscosity, p_y is yield stress, π_c is critical value of π and π is the product of the component of deformation rate with itself, namely, $\pi = e_{ij}e_{ij}$, e_{ij} is the $(i,j)^{th}$ component of deformation rate.

Under the above assumptions the governing flow equations are

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \nu \left[1 + \frac{1}{\beta}\right] \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T \left[T - T_\infty\right] - \frac{\nu}{k^*} u^* \tag{2}$$

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial y^{*2}} - \frac{Q_0}{\rho C_p}\left[T - T_\infty\right] \tag{3}$$

The associated boundary conditions are

$$u^* = u_0\left(1 + \varepsilon e^{in^* t^*}\right), T = T_w \quad at \quad y^* = 0 \tag{4}$$

$$u \to 0, T \to T_\infty \quad as \quad y^* \to \infty \tag{5}$$

From Eq. (1) it is clear that the suction velocity is a function of time only and assuming that it takes the following exponential form:

$$v^* = -V_0\left[1 + A\varepsilon e^{in^* t^*}\right] \tag{6}$$

Here A is a real positive constant, ε and εA are less than unity, and V_0 is a scale of suction velocity which has non-zero positive constant.

We introduce the following dimensionless quantities:

$$u=\frac{u^*}{U_0},\eta=\frac{V_0 y^*}{\nu},t=\frac{t^* V_0^2}{\nu},\theta=\frac{T-T_\infty}{T_w-T_\infty},n=\frac{n^*\nu}{V_0^2},K=\frac{k^* V_0^2}{\nu^2},$$
$$\Pr=\frac{\nu}{\alpha},G_T=\frac{\nu\beta_T g\left[T_w-T_\infty\right]}{U_0 V_0^2},\phi=\frac{\nu Q_0}{\rho C_p V_0^2} \quad (7)$$

In view of (6) - (7), Eqs. (2)-(3) and the boundary conditions (4)-(5) reduces the following form:

$$\frac{\partial u}{\partial t}-\left[1+A\varepsilon e^{\text{int}}\right]\frac{\partial u}{\partial \eta}=\left[1+\frac{1}{\beta}\right]\frac{\partial^2 u}{\partial \eta^2}+G_T\theta-\frac{u}{K} \quad (8)$$

$$\frac{\partial \theta}{\partial t}-\left[1+A\varepsilon e^{\text{int}}\right]\frac{\partial \theta}{\partial \eta}=\frac{1}{\Pr}\frac{\partial^2 u}{\partial \eta^2}-\phi\theta \quad (9)$$

The associated boundary conditions are

$$u=1+\varepsilon e^{\text{int}},\theta=1 \quad at \quad \eta=0 \quad (10)$$

$$u\to 0,\theta\to 0 \quad as \quad \eta\to\infty \quad (11)$$

3. Solution of the problem

The set of Eqs. (8)-(9) represent non-linear partial differential equations (PDE's) that cannot be solved in closed form. However it can reduce to the set of ordinary differential equations (ODE's) in dimensionless form that can be solved analytically. This can be made by exhibiting the velocity and temperature as

$$u=u_0(\eta)+\varepsilon e^{\text{int}}u_1(\eta)+o(\varepsilon^2)+\ldots\ldots\ldots \quad (12)$$

$$\theta=\theta_0(\eta)+\varepsilon e^{\text{int}}\theta_1(\eta)+o(\varepsilon^2)+\ldots\ldots\ldots \quad (13)$$

By using Eqs. (12)– (13) intoEqs. (8)–(9), equating the harmonic and non-harmonic terms, and neglecting the higher-order terms of $O(\varepsilon^2)$, we obtain the following pairs of equations for u_0, u_1, θ_0 and θ_1.

$$\left(1+\frac{1}{\beta}\right)u_1''+u_1'-\left(in+\frac{1}{K}\right)u_1=-Au_0'-Gr\theta_1 \quad (14)$$

$$\left(1+\frac{1}{\beta}\right)u_0''+u_0'-\frac{1}{K}u_0=-Gr\theta_0 \quad (15)$$

$$\theta_1''+\Pr\theta_1'-\Pr\left(in+\phi\right)\theta_1=-\Pr A\theta_0' \quad (16)$$

$$\theta_0''+\Pr\theta_0'-\Pr\phi\theta_0=0 \quad (17)$$

The associated boundary conditions are

$$u_0=1,u_1=1,\theta_0=1,\theta_1=0 \quad (18)$$

The solutions for velocity and temperature fields are

$$u(\eta,t)=u_0(\eta,t)+\varepsilon e^{\text{int}}u_1(\eta,t)$$
$$=\left[B_3e^{-r_3\eta}+B_2e^{-r_1\eta}\right]+\varepsilon e^{\text{int}}\left[B_7e^{-r_4\eta}+B_4e^{-r_3\eta}+B_5e^{-r_2\eta}+B_6e^{-r_1\eta}\right] \quad (19)$$

$$\theta(\eta,t)=\theta_0(\eta,t)+\varepsilon e^{\text{int}}\theta_1(\eta,t)$$
$$=\left[e^{-r_1\eta}\right]+\varepsilon e^{\text{int}}B_1\left[e^{-r_1\eta}-e^{-r_2\eta}\right] \quad (20)$$

The friction factor coefficient and the rate heat transfer coefficients are given by

$$C_f=\left(1+\frac{1}{\beta}\right)\frac{\tau_w}{\rho U_0 V_0}=\left(1+\frac{1}{\beta}\right)\left.\frac{\partial u}{\partial \eta}\right|_{\eta=0} \quad (21)$$

$$Nu=-x\left(\frac{\left(\frac{\partial T}{\partial y^*}\right)_{y^*=0}}{T_w-T_\infty}\right)\Rightarrow Nu\,\mathrm{Re}_x^{-1}=-\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0} \quad (22)$$

4. Results and discussion

The solutions are performed for velocity, temperature, and concentration fields for various values of flow parameters are presented in Figures 1-9 as well as the Skin friction coefficient and the rate of heat transfer coefficient are presented in Tables 1-2. Fig. 1 illustrates the variation of velocity distribution across the boundary layer for different values of the permeability parameter (K). It is noticed that as K increases the velocity profiles increases since the resistance offered by the porous medium decreases as K increases. The velocity profiles for various values of thermal Grashof number is plotted in Fig. 2. It is seen that an increase in Gr leads to a rise in the values of velocity due to improvement in buoyancy force. Here, the positive values of Gr correspond to cooling of the plate. Moreover, it is observed that the fluid velocity increases sharply near the wall of the porous plate as Gr increases and then decays to the free stream value. The impact of the perturbation parameter on velocity profiles are shown in the Figure 3. It is observed that the velocity profiles decreases for increasing values of perturbation parameter. The impact of the Casson parameter on velocity profiles are shown in the Figure 4. It is worthwhile to note that the velocity declines with increasing Casson fluid parameter. Due to the increase of, the yield stress falls and therefore the thickness of momentum boundary layer decreases. The impact of the real positive constant on velocity profiles are shown in the Figure 5. It is observed that the velocity profiles decreases for increasing values of real positive constant. The impact of Prandtl number for the temperature profiles are shown in the Figure 6. The temperature decreases for increasing values of Prandtl number. Figure 7 represents the temperature profiles for various values of heat absorption parameter. It is noticed that as the value of heat absorption parameter increases, the temperature profiles decreases. The impact of the perturbation parameter on temperature profiles are shown in the Figure 8. It is observed that the temperature profiles decreases for increasing values of perturbation parameter. The impact of the Real positive constant on temperature profiles are shown in the Figure 9. It is observed that the temperature profiles decreases for increasing values of real positive constant. From Table 1, with the effect of Casson parameter and Prandtal number, the skin-friction coefficient increases whereas reverse trend is observed in the case of Grashof number, Permeability parameter, heat absorption parameter. From Table 2,

The Nusselt number increases with the increases of heat absorption, Prandtal number and perturbation parameters respectively.

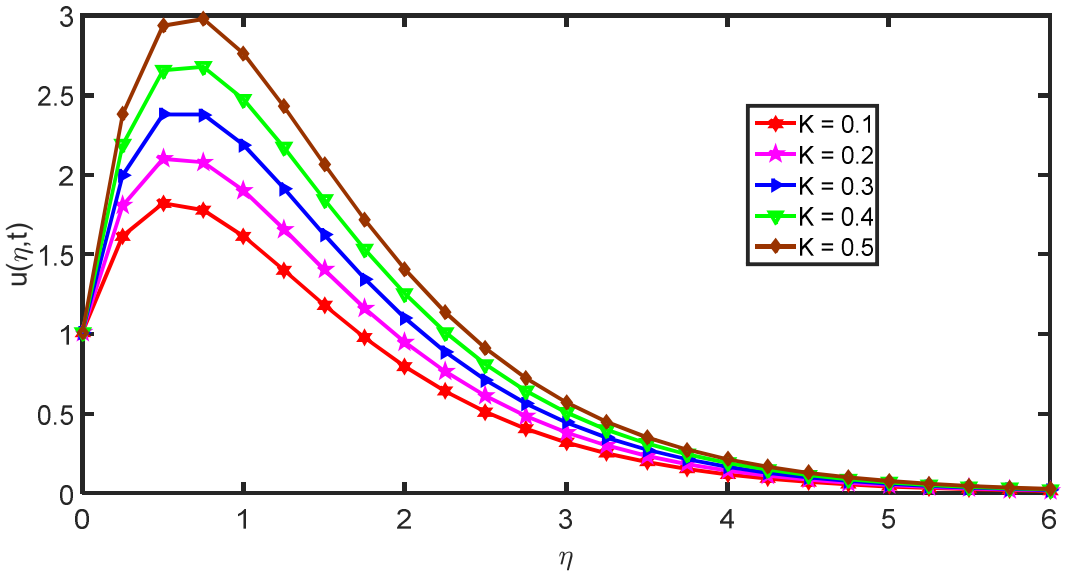

Figure 1. Effect of K on u

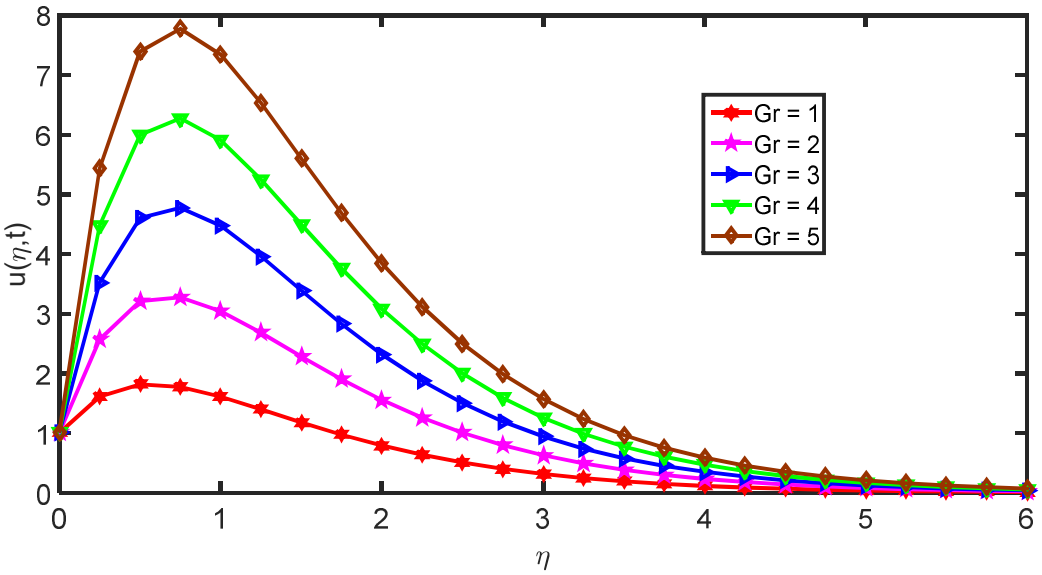

Figure 2. Effect of Gr on u

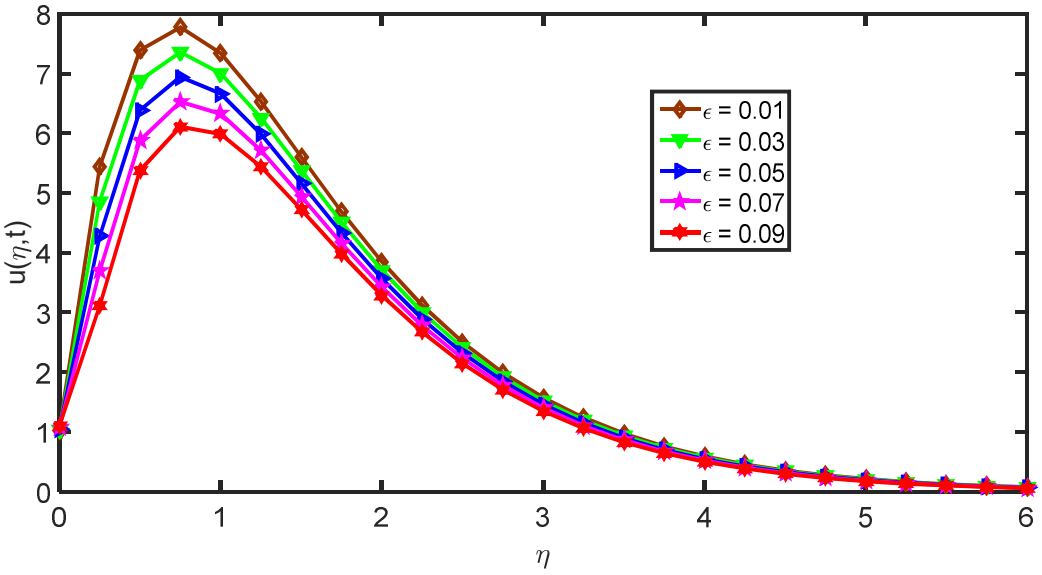

Figure 3. Effect of ε on u

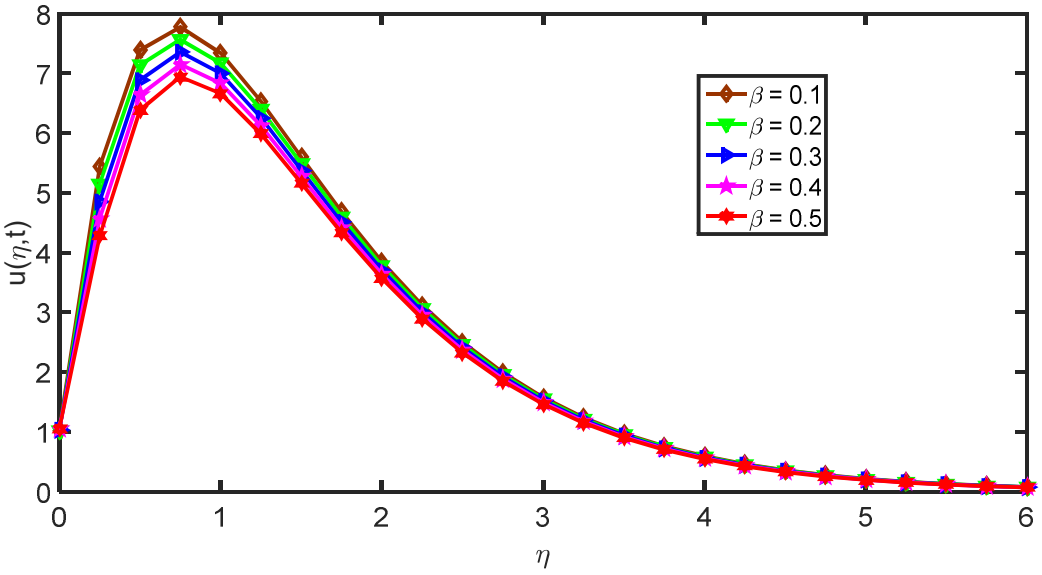

Figure 4. Effect of β on u

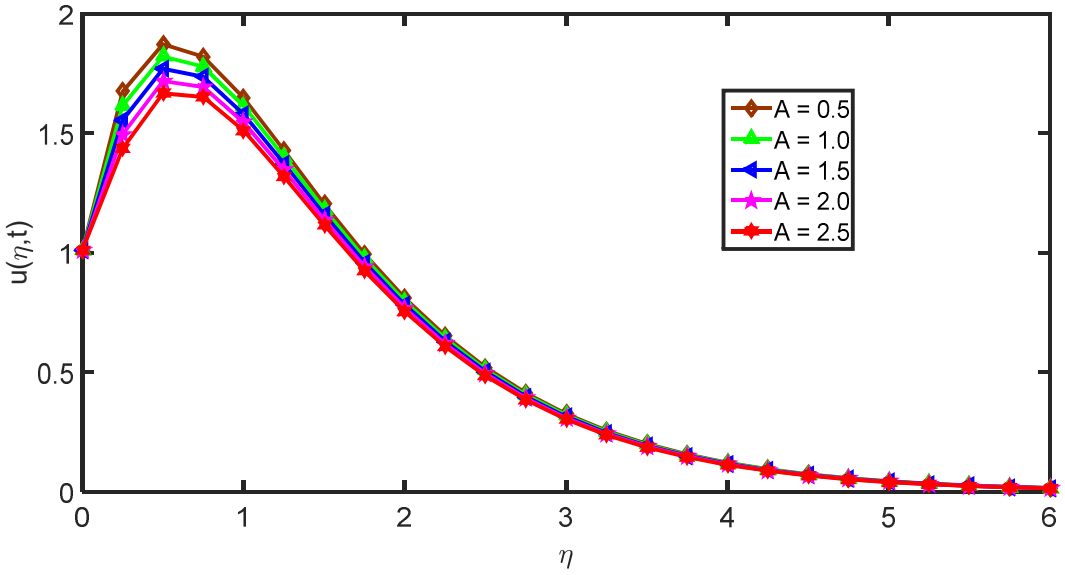

Figure 5. Effect of A on u

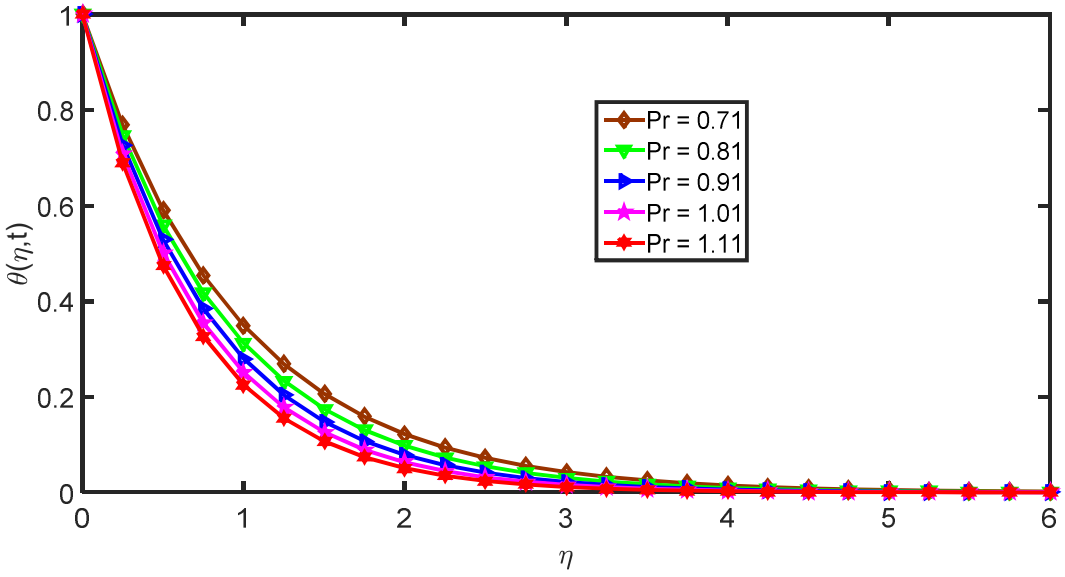

Figure 6. Effect of Pr on θ

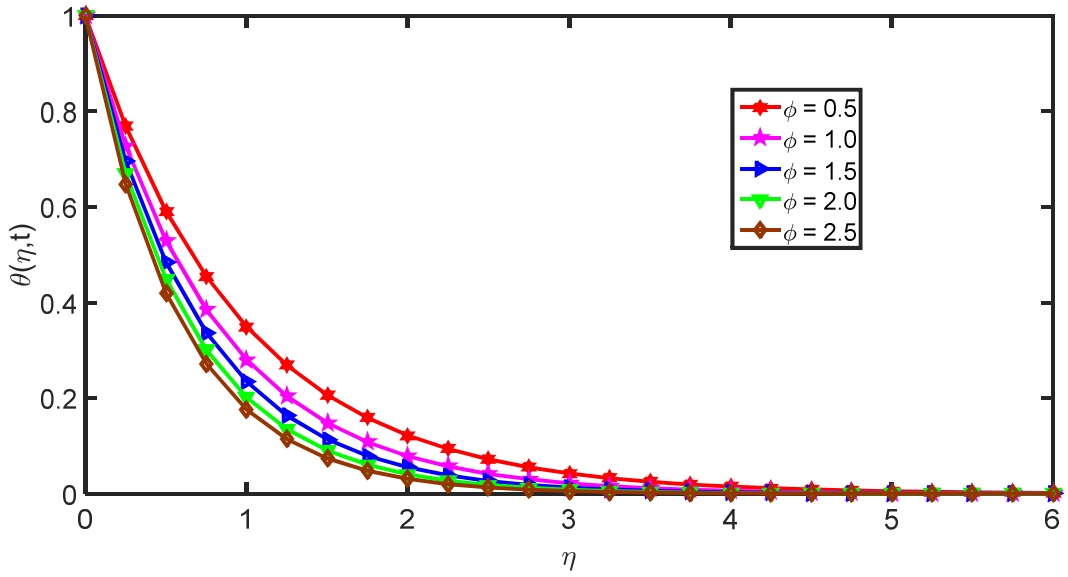

Figure 7. Effect of ϕ on θ

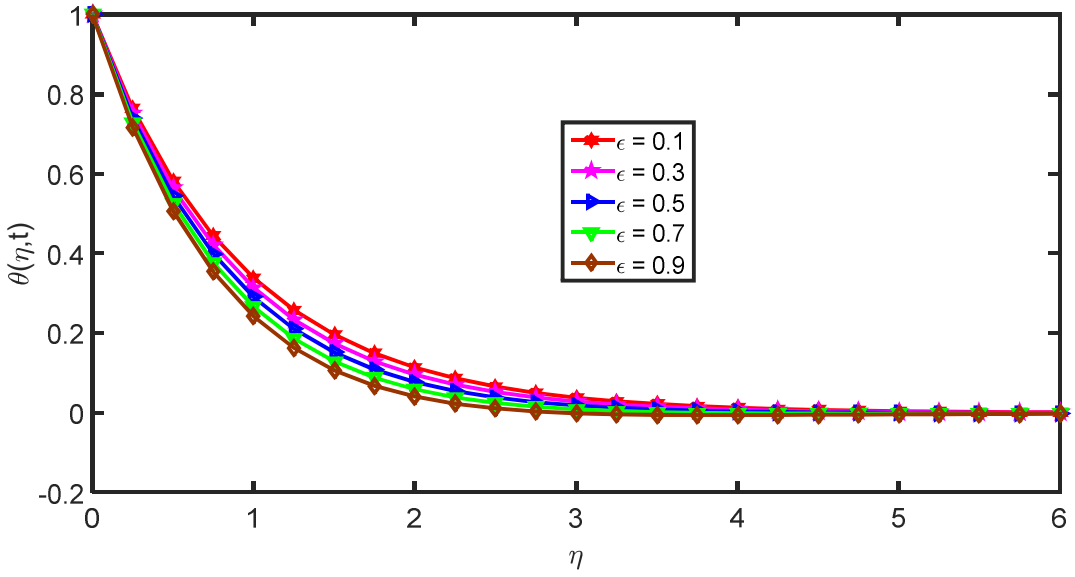

Figure 8. Effect of ε on θ

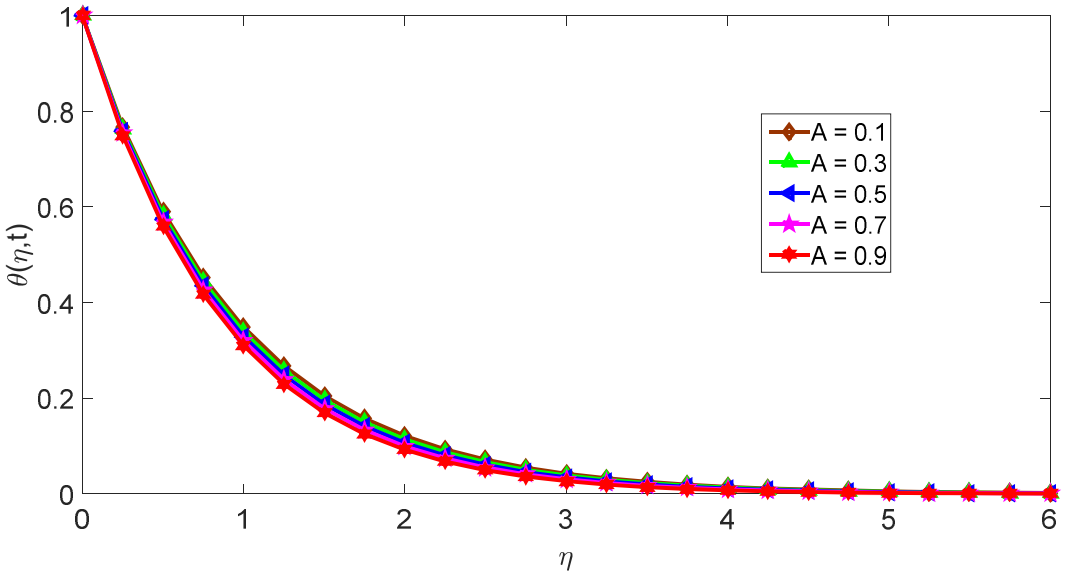

Figure 9. Effect of A on θ.

Table 1. Skin-friction for different values.

Gr	K	β	ϕ	Pr	Cf
1.2					2.0574
1.4					1.6440
1.6					1.2305
1.8					0.8171
	0.1				3.2904
	0.2				1.7873
	0.3				0.8171
	0.4				0.1214
		0.1			1.8113
		0.2			2.9249
		0.3			4.1455
		0.4			6.8751
			0.1		0.8062
			0.2		0.7965
			0.3		0.7919
			0.4		0.7803
				0.1	0.7847
				0.2	1.4724
				0.3	1.9986
				0.4	2.3635

Table2. Nusselt number for different values.

ϕ	Pr	ε	Nu
0.1			0.8020
0.2			0.8757
0.3			0.9401
0.4			0.9981
	1		1.3700
	2		2.4227
	3		3.4498
	4		4.4676
		0.05	1.0619
		0.10	1.0753
		0.15	1.0887
		0.20	1.1020

5. Final Remarks

In this work, an unsteady heat transfer Casson fluid flow over a vertical plate with heat absorption effect was investigated. The main conclusions of this study are as flows:

- The momentum boundary layer thickness increases with enhancing values of β and Pr.
- The temperature of the fluid is a decreasing function of A, Pr & ϕ.
- With the effect of Casson parameter and prandtal number, the skin-friction coefficient increases whereas reverse trend is observed in the case of Grashof number, Permeability parameter, heat absorption parameter.
- The Nusselt number increases with the increases of heat absorption, prandtal number and perturbation parameters respectively.

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