BUTTERFLY GRAPH AND ITS ACHROMATIC NUMBER

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Abstract:

. In this paper, we find the achromatic number of three copies of **barbell** graph with their achromatic number. The largest possible number of colours in an achromatic colouring is called the achromatic number and is denoted by $\chi_a(G)$,where G is a finite un directed graph with no loops and multiple edges.

Key words:

Barbell graph, butterfly graph , Achromatic number.

Introduction

The achromatic number was defined and studied by Harary, Hedetniemi and prins. Yannakakis and Gavril Proved that determining this value for general graphs is NP –complete. The NP-completeness of the achromatic number for trees was established by Cairnie and Edwards.

Further it is solvable for paths , cycles, complete bipartite graphs. Roichman, gives the achromatic number for Hyper cubes. The achromatic number was defined and studied by Harary and Hedetniemi. Computing the achromatic number for a general graph was proved NP-complete by Yannakakis and Gavril.

A simple proof of this fact appears in Bodlaender proved , further, that the problem remains NP-complete even when we limit ourselves to connected graphs that are both interval graphs and co-graphs. The NP-completeness of the achromatic number for trees was established only recently. For graphs that are complements of trees this problem can be solved in polynomial time.

DEFINITIONS

1.Vertex Colouring

A k-vertex colouring of a graph G, or simply a k- colouring is an assignment of k colours to its vertices. The colouring is proper if no two adjacent vertices are assigned the same colour.

A graph is k- colourable if it has a proper k- colouring.

2.Edge Colouring

A k-edge colouring of a graph G, or simply a k- colouring is an assignment of k colours to its edges. The colouring is proper if no two adjacent edges are assigned the same colour.

A graph is k-edge colourable if it has a proper k-edge colouring.

3.Chromatic Number

The chromatic number of a graph G is the least k for which G is k-vertex colourable and it denoted by $\chi(G)$ .A graph G is k-chromatic if $\chi(G)$ = k.

The chromatic number of a graph G is the least k for which G is k-vertex colourable and it denoted by $\chi'(G)$ .A graph G is k-chromatic if $\chi'(G)$ = k.

4. Line Distinguishing Colouring

Let G(V,E) be a graph. A colouring $\phi : V \rightarrow N$ of the vertices is a line distinguishing colouring iff for every edge (u, v) ε E the edge colour (ϕ (u), ϕ(v)) is unique,(i.e). it appears at most once.

5 Achromatic Colouring and achromatic Number

The achromatic colouring of a graph is a proper vertex colouring such that each pair of colour classes is adjacent by at least one edge. The largest possible number of colours in an achromatic colouring of a graph G is called the and it is denoted by $\chi_a(G)$.

6. Pseudo Achromatic Number

The pseudo achromatic number α (G) is the maximum k for which there exists a complete colouring of G. If the colouring is required also to be proper , then such a maximum is known as the achromatic number and it will be denoted here by $\chi_a(G)$.

7. Two copies of Barbell graph

Two copies Barbell graph is the simple graph obtained by connecting two copies of a complete graph G_1, G_2 by a bridge and it is denoted by $B(G_1, G_2)$.

8. Three copies of Barbell graph

Three copies of Barbell graph is the simple graph obtained by connecting three copies of a complete graph G_1, G_2, G_3 by a bridge and it is denoted by $B(G_1, G_2, G_3)$.

***9*. N copies of Barbell graph**

Barbell graph is the simple graph obtained by connecting N copies of a complete graph K_n by a bridge and it is denoted by $B(G_1, G_2, G_3, \ldots G_N)$.

10. Butterfly networks

Undirected graphs whose nodes represented as processors and edge represented as inter processor communication links are defined by networks. For example the following fig. 1 represents two dimensional butterfly network BF(2).

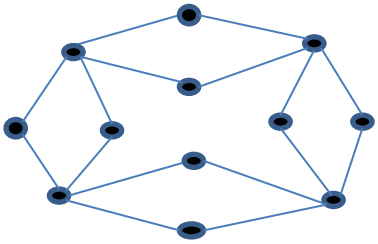

Figure 1 Two dimensional butterfly network

Properties of Achromatic Colouring

Lower Bound

In general graphs it is difficult to find lower bound. For particular graphs with large girth (at least 5) admit algorithms with relatively low approximation ratio for the achromatic number. This result gives on the observation that $\chi_a(G) \leq$ m/n for graphs G with n vertices, m edges and girth at least 5.

Upper Bound

In general graphs it is difficult to find upper bound. For a particular case approximating the achromatic number for general or bipartite graphs, the approximation ratio guarantees are just barely sub-linear in the number of vertices.

NP-complete and NP hard

The problem of achromatic colouring is NP-complete for general graphs. The problem of achromatic colouring is NP-hard for trees, bipartite graphs, interval graphs, bipartite permutation and quasi-threshold graphs.

Theorem

We find the following result for 3 copies of barbell graph and extended to N copies of barbell graph by our definition (7) .

Theorem 1:

For any complete graph BF(2), N copies of achromatic colouring is χ_a [B(BF(2),BF(2),BF(2)]=12*3- 4*3=24, n≥ 2

***Proof*:**

Let G= B(K_n,K_n ,K_n) be the Barbell graph. By the definition, (3,n) Barbell graph is obtained by connecting three copies of complete graph K_n by a bridge. Let V = {v_1, v_2 , v_3..... ,v_{n-4}} be the vertex set of K^1 ,W={ w_1, w_2 , w_3..... ,w_{n-4}} be the vertex set of K^2 and X=={ x_1, x_2 , x_3..... ,x_{n-4}} be the vertex set of K^3 .

Now the colouring assignments are as follows. Since K^1 contains exactly 'n' vertices (n ≥2) which are mutually adjacent to each other, we should colour the outer vertices using n-4 colours and the remaining inner vertices of 'K^1' by n different colours B_1^i ,where i=1,2….n. For colouring 'K^2', take from 'K^2' & 'K^3'. We should use outer 'n-4' different colours C_1^i apart from B_1^i and any four colour from B_1^i for inner vertices,1≤ i ≤n.

Similarly colour 'K^3', we should use 'n-4' different colours A_1^i apart from C_1^i and any four colour from C_1^i or B_1^i ,1≤ i ≤n. Thus the number of colours required is equal to (n-4)+(n-4)+(n-4)=3n-12.

Example:1

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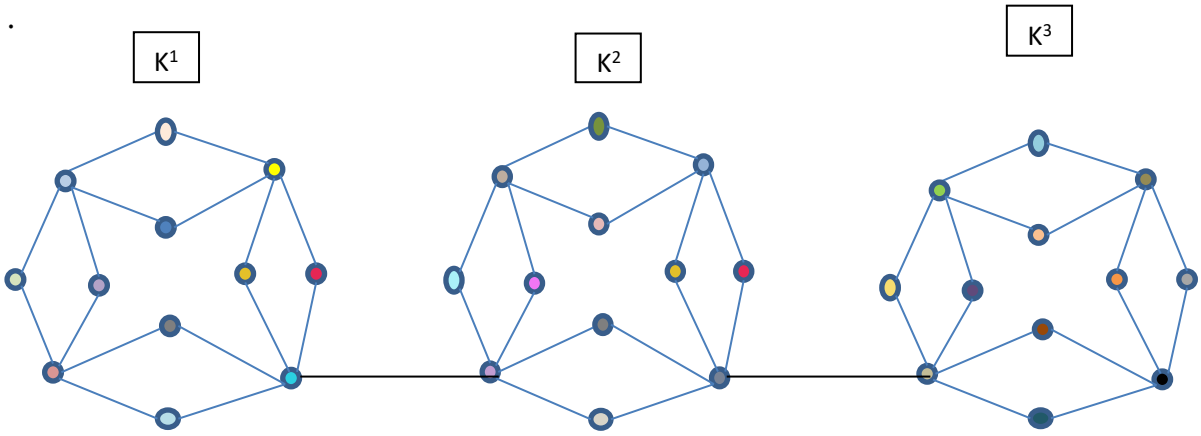

Figure 1. χ_a [B(BF(2),BF(2),BF(2)]=12*3- 4*3=24, n≥ 2

Theorem 2:

For any complete graph BF(2), N copies of achromatic colouring is χ_a [B(BF(2),BF(2),….BF(2)]=(n-4)N, N ≥ 2

Conclusion:

We constructed the theorem for three copies of barbell graph and its achromatic number of butterfly graphs. Further research work we were extended to N copies of barbell graph of complete graphs.

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